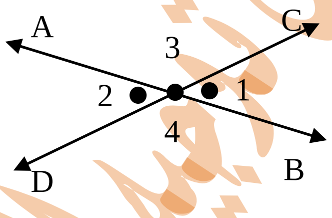
Geometry and measurement

Deductive Proof

Theorem (1):

- If two straight lines intersect, then the measure of each two vertically opposite angles are equal.



Given: The two straight lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at E

Required to proof

R.T.P.

m (∠1)= m (∠2)

Proof:

∵ ∠2 and ∠3 are adjacent angles where $\overrightarrow{ED} \cup \overrightarrow{EC} = \overleftrightarrow{CD}$

∴ m (∠2) + m (∠3) = 180°

∵ ∠2 and ∠ 4 are adjacent angles where $\overrightarrow{EA} \cup \overrightarrow{EB} = \overleftrightarrow{AB}$

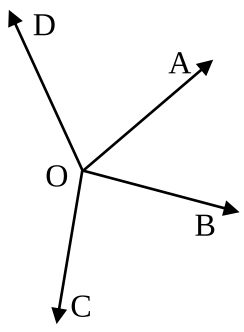
∴ m (∠2) + m (∠4) = 180°

∵ m (∠2) + m (∠3) = m (∠2) + m (∠4)

∴ **m (∠3) + m (∠4)**

Q.E.D. Also: **m (∠1) = m (∠2)**

Theorem (2): The sum of the measure of the accumulative angles at a point is equal to 360°



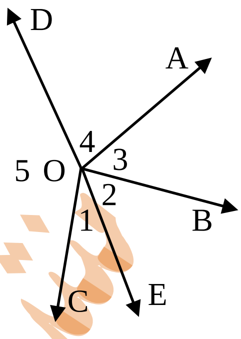
Given: $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ are rays that start at O

R.T.P. Prove that the sum of the measure of the accumulative angles at the point Ois 360°

Construction: Extend $\overrightarrow{DO}$ to produce E.

Proof:

∵ m (∠2) + m (∠3) + m (∠4) = 180°



$\therefore$ m $(\angle 1)$ + m $(\angle 5) = 180^o$

$\because$ m$(\angle 1)$ + m$(\angle 2)$ + m$(\angle 3)$ + m$(\angle 4)$ + m$(\angle 5)$

$= 180^o + 180^o = 360^o$

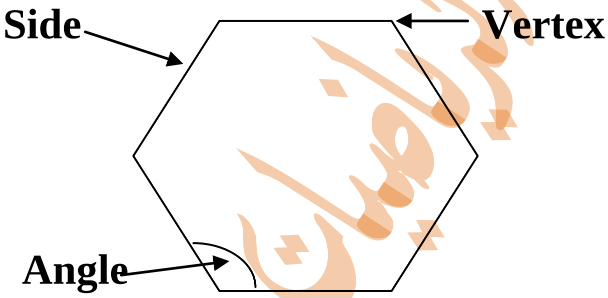
$\therefore$ **m $(\angle 3)$ + m $(\angle BOC)$ + m $(\angle 5)$ + m $(\angle 4) = 360^o$**

Q.E.D.

The polygons

Definition:

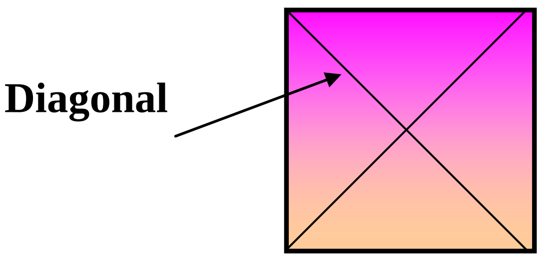
The polygon is a simple closed line formed from three line segments or more.



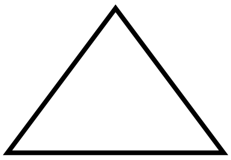
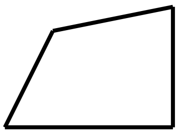
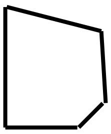
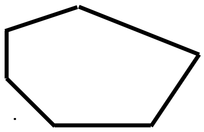
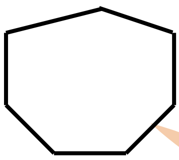
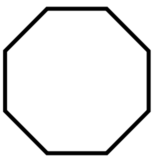
Important notes

(1) The number of **sides** of any polygon is equal to the number of its **vertices** and equal to the number of its **angles.**

(2) The diagonal is the line segment joining two non-consecutive vertices.



The polygon is named after the number of its sides as following:

Polygon	Number Of sides	Number Of vertices	Number Of angles
 Triangle	3	3	3
 Quadrilateral	4	4	4
 Pentagon	5	5	5
 Hexagon	6	6	6
 Heptagon	7	7	7
 Octagon	8	8	8

Note:

You can get the number of diagonals of n–sides polygon from the rule:

$$\frac{n\,(n-3)}{2}$$

, where n is the number of sides.

Rules:

(1) The sum of measures of the **interior** angles of **n-sides** polygon

$= (n - 2) \times 180°$, where n is the number of sides.

(2) The sum of measures of the **exterior** angles of **any polygon** = 360°

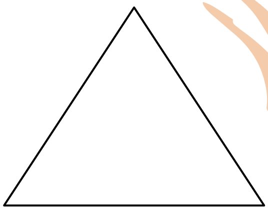
Note that :

1) The sum of measures of the interior angle and the exterior angle at any vertex is 180°.
2) If one or more of the angles of the polygon is reflex, then the polygon is concave.
3) If none of the angles of the polygon is reflex, then the polygon is convex.

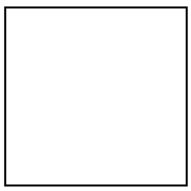
Regular Polygon

The regular polygon is the polygon in which :

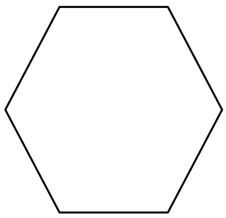
- **All of its sides are equal in length.**
- **All of its angles are equal in measure.**



Equilateral triangle



Square



Regular Hexagon

Important notes:

1) Perimeter of regular polygon = length of its side × number of sides
2) The measure of **<u>each interior angle</u>** of a regular polygon :

$$= \frac{(n-2) \times 180°}{n}$$, where n is number of sides (angles).

<u>Remark:</u> The number of sides of any polygon = $\frac{360°}{180° - X}$

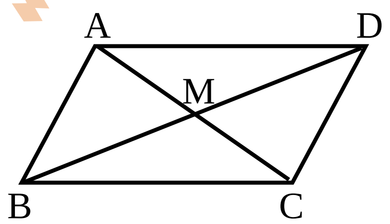
Where X is the measure of the interior angle of this polygon.

The parallelogram and its properties

<u>Definition</u>:

A parallelogram is a quadrilateral in which each two opposite sides are parallel.

→ **<u>The properties of a parallelogram</u>**:



1) Each two opposite sides are equal

in length and parallel.

$\overline{AB}$ // $\overline{CD}$, $\overline{AD}$ // $\overline{BC}$

AB = CD , AD = BC

2) Each two opposite angles are equal in measure

m (<A) = m (<C) , m (<B) = m (<D)

3) The sum of measures of each two consecutive angles in a parallelogram is 180^0

m (<A) + m(<B) = 180^0 , m (<C) + m (<D) = 180^0

m (<B) + m(<C) = 180^0 , m (<A) + m (<D) = 180^0

4) The diagonals bisect each other

AM = MC , BM = MC

The special cases of the parallelogram

[1] The rectangle:

Is parallelogram with a right angle

Properties of rectangle:

1) The four angles of a rectangle are all equal in measure and the measure of each is 90^0.
2) The two diagonals are equal in length.

[2] The rhombus:

Is parallelogram in which each two adjacent sides are equal in length.

1) The four sides are all equal in length.
2) The two diagonals are perpendicular and bisect each of its interior angles.

[3] The square:

Is a parallelogram with a right angle and two adjacent sides are equal in length.

The properties of the square:

1) All sides are equal in length.
2) All angles are equal in measure and each one = 90^0
3) The diagonals are equal in length, perpendicular and each diagonal bisects the two vertices angles which this diagonal joins.

We can also define the square as follows:

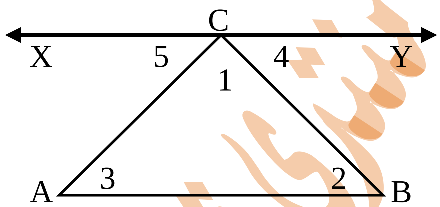
1) A square is a rectangle with two adjacent sides equal in length.
2) A square is a rhombus with a right angle.

Triangle properties

Theorem (1): The sum of the measure of the interior angles of a triangle is 180°

Given: ABC is a triangle

R.T.P. m (∠1) + m (∠2) + m (∠3) = 180°



Construction: Draw $\overleftrightarrow{XY}$ // $\overline{AB}$ passing through the point C.

Proof: ∠XCY is a straight angle = 180°

∴ **m (∠1) + m (∠4) + m (∠5) = 180°** → (*)

∵ $\overline{XY}$ //$\overleftrightarrow{AB}$

∴ m (∠5) = m (∠3) alternate angles → (1)

m (∠4) = m (∠2) alternate angles → (2)

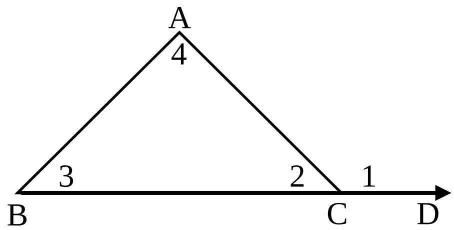
Replace (1) and (2) in (*)

∴ **m (∠1) + m (∠2) + m (∠3) = 180°** **Q.E.D.**

Theorem (2):

- The measure of the exterior angle of a triangle is equal to the sum of the two non- adjacent interior angles.
- ∠2 , ∠3 and ∠4 are called interior angles

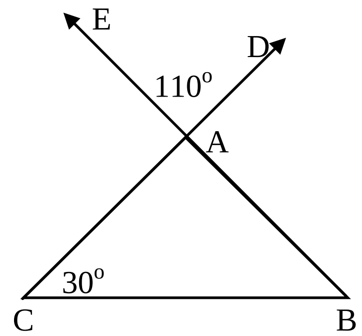
Of ΔABC ∠1 is called exterior angle



Then: m (∠1) = m (∠3) + m (∠4).

Example: m (∠DAE) = 110^0 , m (∠C) = 30°

Find m (∠B) and m (∠EAC)



Given

m (∠C) = 30° and **m (∠DAE) = 110°**

R.T.P. m (∠B) and m (∠EAC)

Proof:

$\because$ m ($\angle$DAE) = 110°

$\because$ m ($\angle$DAE) = m ($\angle$BAC) Vertically opposite angles

$\because$ m($\angle$BAC) = 110°

$\therefore$ The sum of measures of the interior angles of a triangle = 180°

$\therefore$ m ($\angle$B) = $180^{o} - (110^{o} + 30^{o}) = 40^{o}$

Required (1)

$\because \angle$ EAC is an exterior angle of ΔABC

$\therefore$ **m (EAC) = m ($\angle$B) + m ($\angle$C) = $30^{0} + 40^{0}$**

m ($\angle$EAC) = 70°

The triangle (2)

Theorem (2)

The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.

Corollary

The line segment joining the midpoints of two sides of a triangle is parallel to the third side.

Theorem (3)

The length of the line segment joining the midpoints of two sides of a triangle is equal to half the length of the third side.

Important notes

1-The measure of the exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

2-If two angles of one triangle equal two angles of another triangle in measure, then the third angle of the first triangle is equal in measure to the third angle of the other triangle.

3-If the sum of measures of two angles in a triangle equal 90^0, then the third angle is right.

4-If the sum of measures of two angles in a triangle is less than 90^0, then the third angle is obtuse.

5–If the sum of measures of two angles in a triangle is more than 90^0, then the third angle is a cute.

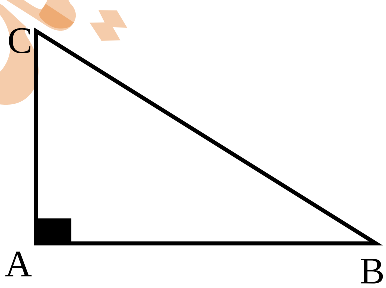
Converse of Pythagoras's theorem

In a triangle, if the sum of the areas of two squares on two sides is equal to the area of the square on the third side, then the angle opposite to this side is a right angle.

→ In Δ ABC;

if $(AB)^2 + (AC)^2 = (BC)^2$

then: $m(\angle A) = 90^0$



Corollary:

In ΔABC, if $\overline{AC}$ is the longest side and if $(AC)^2 \neq (AB)^2+(BC)^2$, then $m(\angle B) \neq 90^0$ and the triangle is not right–angled.

Questions

Model (1)

(1) Choose the correct answer:

1) The sum of the measure of interior angle of a triangle equals:

a) 90°   b) 180°   c) 270°   d) 360°

2) The image of the point (-1 , 3) by translation (4 , -2) is ……………

a) (3 , 1)   b) (3 , -1)   c) (5 , 1)   d) (5 , -5)

3) The measure of the exterior angle of the equilateral triangle is ……

a) 30°   b) 45°   c) 60°   d) 120°

4) In a parallelogram if the two adjacent sides are equal in the length, then the shape is ……….

a) square   b) rhombus   c) rectangle   d) trapezium

5) The number of the diagonals of a pentagon is …………

a) 3   b) 5   c) 7   d) 9

(2) Complete:

1) The image of the point (2 , 1) by reflection in X-axis is ……………

2) The image of the point (2 , -1) by rotation about the origin point with an angle of measure 180° is ……………

3) The square is a rectangle in which ……………..

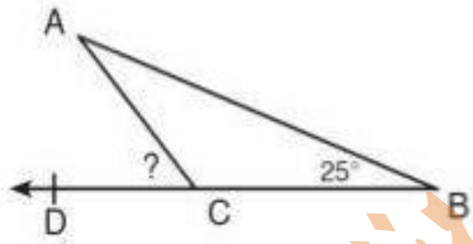
4) ABCD is parallelogram in which m (∠ A) = 60°, then m (∠ B) = ……

5) The image of the point (5 , 3) by translation: (x , y) → (x + 3 , y - 1) is ……………..

(3) a) In the opposite figure:

m (∠ A) = m (∠ B) = 25°

Find m (∠ ACD)

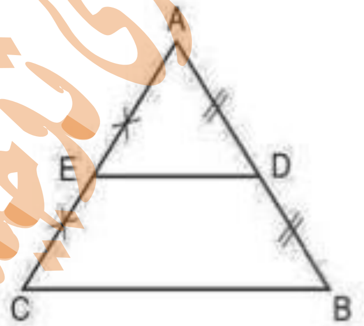


b) In the opposite figure:

Δ ABC in which: AB = 12 cm,

BC = 10 cm, AC = 8 cm

Find the perimeter of Δ ADE

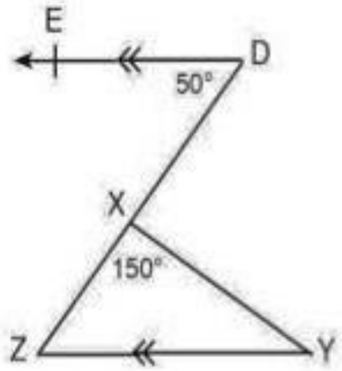


(4) a) In the opposite figure:

$\overrightarrow{DE}$ // $\overline{YZ}$, m (∠ ZDE) = 50°

, m (∠ YXZ) = 105°

Find: m (∠ Z) , m (∠ Y) , m (∠ YXD)

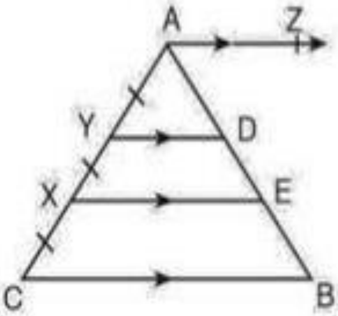


b) In the opposite figure:

$\overrightarrow{AZ}$ // $\overline{YD}$ // $\overline{XE}$ // $\overline{CB}$

AY = YX = XC, AB = 18 cm,

Find the length of $\overline{EB}$

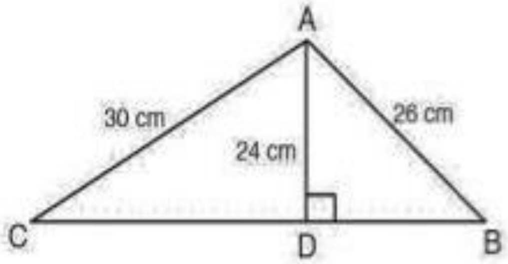


(5) a) In the opposite figure:

$\overline{AD}$ ⊥ $\overline{BC}$, If AD = 24 cm , AB = 26 cm,

AC = 30 cm, find the length of $\overline{BC}$ then

find the area of Δ ABC



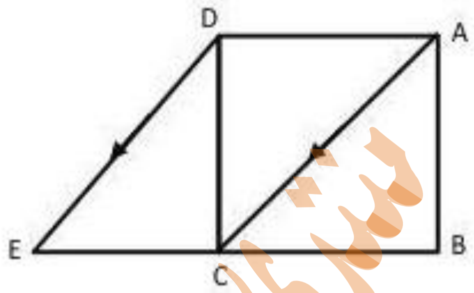
b) In the opposite figure:

ABCD is a square. E ∈ $\overrightarrow{BC}$

$\overline{AC} \parallel \overline{DE}$

Prove that:

ACED is a parallelogram



Model (2)

(1) Choose the correct answer:

1) The image of the point (2 , -5) by reflection x-axis is ……………

a) (2 , -5) b) (2 , 5) c) (-2 , -5) d) (5 , 2)

2) The measure of each angle of regular hexagon equals …………

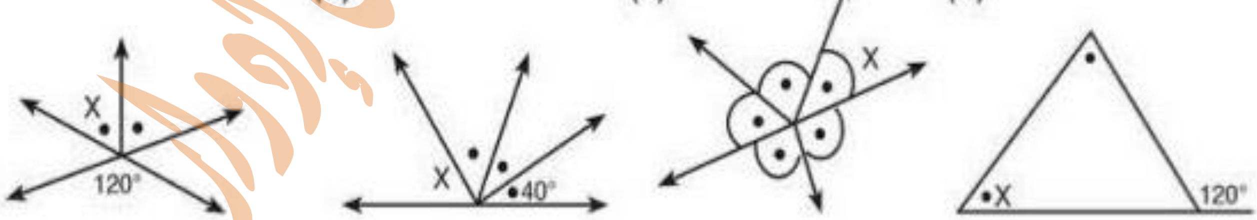
a) 60° b) 108° c) 120° d) 135°

3) The two diagonals are equal in the length and not perpendicular in ………….

a) parallelogram b) rectangle c) rhombus d) square

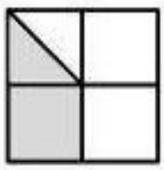
4) All the following shapes m (∠ X) = 60° except the shape ……….

(a) (b) (c) (d)



5) In the opposite figure:

The area of shaded part from the area of all shape equal



a) $\frac{1}{8}$ b) $\frac{1}{4}$ c) $\frac{3}{8}$ d) $\frac{3}{4}$

(2) Complete:

1) The sum of the measure of the angles of the quadrilateral equals ....°

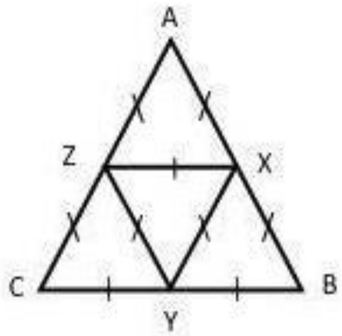
2) The image of the point (2 , 3) by translation $\overrightarrow{MN}$, in direction MN, where m (2 , -1) , N (5 , 1) is ....

3) ABCD is parallelogram in which m (∠ A) = 60°, then m (∠ B) = ...

4) The ray drawn parallel to one side of a triangle and passing through the mid-point of another side ....

5) In the opposite figure:

The image of the triangle XBY by translation XZ in direction $\overrightarrow{XZ}$ is ....

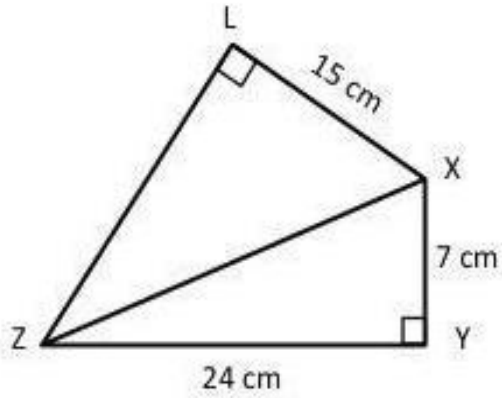


(3) a) In the opposite figure:

XYZL is quadrilateral in which m (∠ Y) = m (∠ L) = 90° , XY = 7 cm

YZ = 24 cm, XL = 15 cm.

Find the length of each of $\overline{XZ}$, $\overline{LZ}$



b) Using the square lattice, draw $\overline{AB}$ where A (4 , 3) , B (-1 , 1) then find the image of $\overline{AB}$ by transition (x , y) → (x + 2 , y - 1)

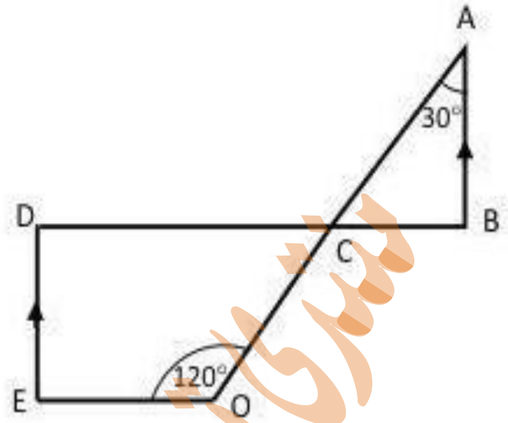
(4) a) Draw the image of triangle ABC where A (1 , 1) , B (3 , 4) , C (5 , 2) by reflection in X-axis

b) In the opposite figure:

$\overline{AB}$, $\overline{ED}$ are perpendicular to $\overline{BD}$, $\overline{BD} \cap \overline{AO} = \{ C \}$

m (∠ A) = 30°, m (∠ EOC) = 120°

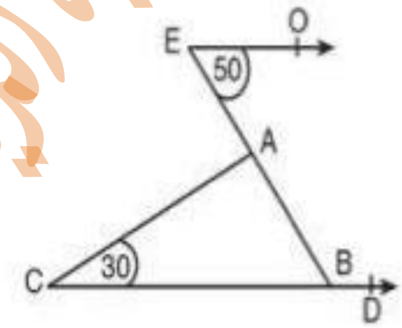
Find m (∠ E)



(5) a) In the opposite figure:

$\overrightarrow{EO} \parallel \overline{CD}$, m (∠ E) = 50°

, m (∠ C) = 30°, find the measures of angles of Δ ABC , m (∠ ABD)



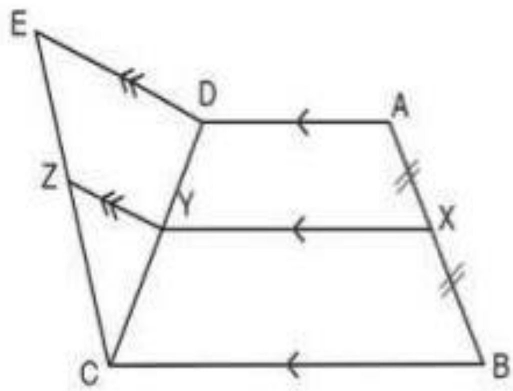
b) In the opposite figure:

X is the midpoint of $\overline{AB}$

, Y ∈ $\overline{CD}$, Z ∈ $\overline{CE}$

, $\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$, $\overline{YZ} \parallel \overline{DE}$

Is CZ = ZE? giving reason.



Model (3)

(1) Choose the correct answer:

1) The image of the point (3 , -5) by reflection in y-axis is ....

a) (3 , 5) b) (-3 , -5) c) (-3 , 5) d) (-5 , 3)

2) The sum of the measure of interior angles of a pentagon is ......

a) 360° b) 450° c) 540° d) 720°

3) The number of diagonals of quadrilateral is ....

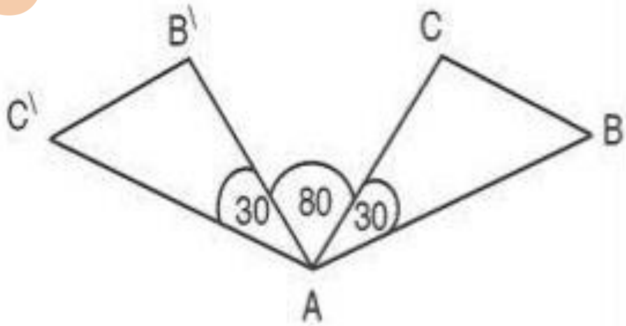
a) 2 b) 3 c) 4 d) 5

3) The diagonal of a square divided its vertex angle in two angles of the measure of each of them is ....

a) 30° b) 45° c) 60° d) 90°

4) In the opposite figure:

Δ AB`C` is the image of Δ ABC by rotation about A and with angle of measure ....



a) 30° b) 80°

c) 110° d) 140°

5) The diagonal of a square divided its vertex angle in two angles of the measure of each of them is ....

a) 30° b) 45° c) 60° d) 90°

(2) Complete:

1) The rhombus is a parallelogram in which ....

2) Each two opposite angles in a parallelogram are ....

3) (-3 , 2) is the image of the point (3 , 2) by reflection in .... axis.

4) The line segment joining the midpoint of two sides of a triangle is …

5) The image of the point (4 , 6) by geometric transformation

$(x , y) \rightarrow (-x , y - 7)$ is ………….

(3) a) Using the lattice, draw △ ABC where A (1 , 0) , B (0 , 2) and C (-3 , 1), then draw its image by reflection in X-axis.

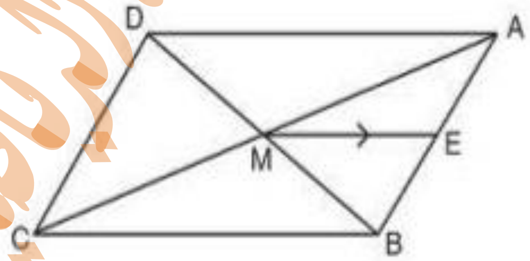
b) In the opposite figure:

ABCD is a parallelogram,

M is the intersection of its diagonals

Draw $\overrightarrow{ME} \parallel \overline{CB}$

Is AE = EB? giving reason.



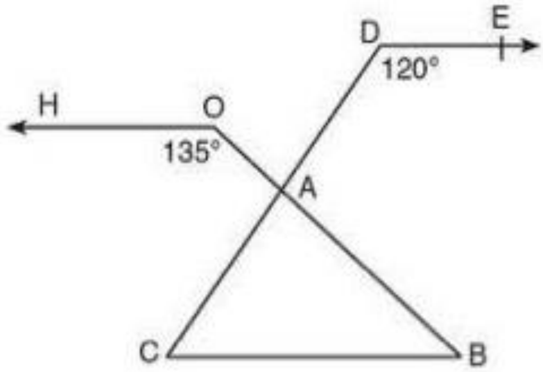
(4) a) In the opposite figure:

$\overrightarrow{DE} \parallel \overrightarrow{OH} \parallel \overline{BC}$, m (∠ ADE) = 120°

, m (∠ AOH) = 135°.

Calculate the measures

of the angles of the triangle ABC

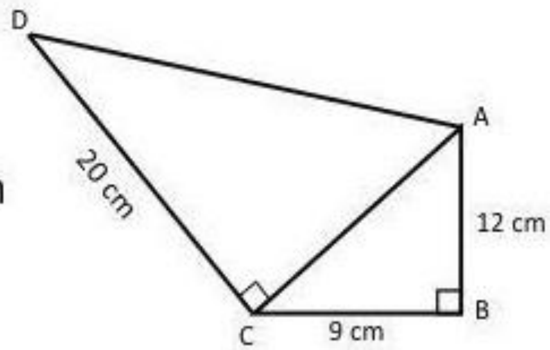


b) In the opposite figure:

m (∠ B) = m (∠ ACD) = 90°,

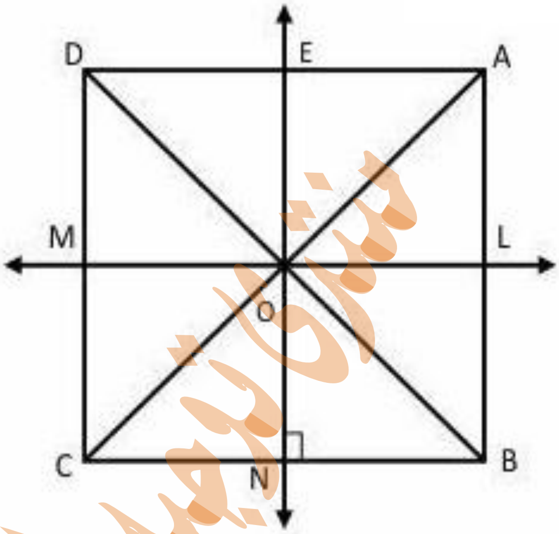
AB = 12 cm, BC = 9 cm and AD = 20 cm

Find the length of each $\overline{AC}$ and $\overline{CD}$



(5) In the opposite figure:

ABCD is a square of side length 6 cm and the origin point its center. Find:



a) The image of Δ AOL by transition 3 cm in direction $\overrightarrow{AB}$

b) The image of Δ AOL by reflection in $\overleftrightarrow{EN}$.

c) Find the image of Δ AOL by rotation about O and with an angle of measure (-90°)

Model (4)

(1) Choose the correct answer:

1) The parallelogram whose diagonals are perpendicular and not equal in length is called ....

a) rhombus b) square c) rectangle d) trapezium

2) The measure of each angle of a regular pentagon is ....

a) 90° b) 108° c) 120° d) 136°

3) The triangle contains at least two .... angles.

a) acute b) obtuse c) rights d) reflex

4) If ABCD is a parallelogram in which BC = 8 cm, CD = 6 cm, then its perimeter = ...

a) 14 cm b) 28 cm c) 48 cm d) 56 cm

5) The image of the point (2 , -1) by reflection in x-axis is ....

a) (2 , 1) b) (1 , 2) c) (-2 , -1) d) (-1 , 2)

(2) Complete:

1) The line segment joining the midpoints of two sides of a triangle ....

2) In a parallelogram each two opposite angles are ....

3) The quadrilateral is a parallelogram if ....

4) The image of the point (5 ,-3) by translation 3 units in negative direction of x-axis is ....

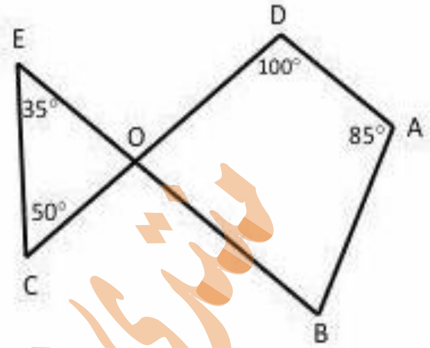
5) The image of the point (3 , 2) by rotation with an angle of measure 180°about the origin is ....

(3) In the opposite figure:

$\overline{DC} \cap \overline{BE} = \{ C \}$,m (∠ A) = 85°

, m (∠ D) = 100°, m (∠ E) = 35°, m (∠ C) = 50°

Find with proof each of m (∠ DOB) , m (∠ B)



(4)

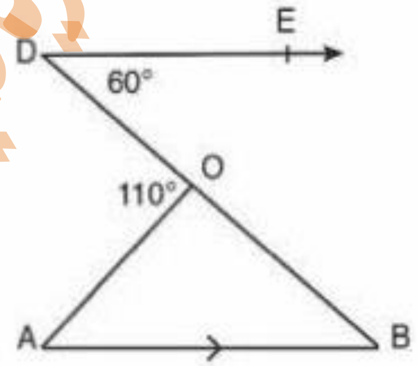
a) Find the length of the diagonal of a rectangle whose area 48 cm^2 and of width 6 cm?

b) In the opposite figure:

$\overline{AB} // \overrightarrow{DE}$, m (∠ D) = 60°,

m (∠ AOD) = 110°

Find with proof m (∠ A)



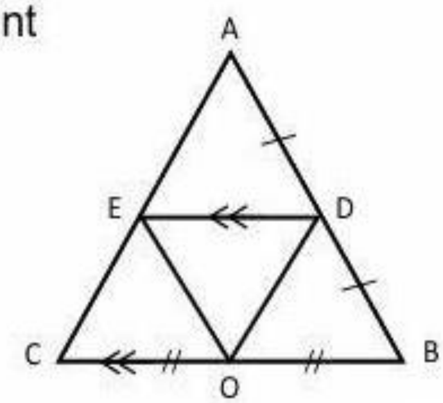
(5)

a) If the point A is the image of point B (-1 , 2) by reflection in y-axis, find the image of A by translation (-1 , 2)

b) In the opposite figure:

ABC is a triangle in which D is the midpoint of $\overline{AB}$, O is the midpoint of $\overline{BC}$, E ∈ $\overline{AC}$ such that $\overline{DE} // \overline{BC}$, AB = 10 cm , BC = 12 cm , AC = 8 cm

Prove that DBOE is a parallelogram, then find the perimeter of Δ EDO



Model (5)

(1) Choose the correct answer:

1) The image of the point (-3 , 4) by reflection in y-axis is ....

a) (3 , -4) b) (3 , 4) c) (-3 , -4) d) (4, -3)

2) The sum of the measure of the exterior angles of a triangle equals ...

a) 90° b) 108° c) 180° d) 360°

3) The diagonals which are equal in the length and perpendicular in ....

a) square b) rhombus c) rectangle d) parallelogram

4) The image of the point (-3 , 5) by rotation about the origin and with an angle of measure 90° ....

a) (5 , 3) b) (-5 , 3) c) (3 , 5) d) (-5 , -3)

5) The measure of each angle of regular octagon equals ....

a) 108° b) 120° c) 135° d) 144°

(2) Complete:

1) The parallelogram whose diagonals are perpendicular is ....

2) If the measure of an interior angle of a triangle is equal to the sum of the measures of the other two interior angles, then the triangle is ....

3) Any triangle has at least two .... interior angles.

4) The image of the point (2 , -4) by reflection in x-axis is ....

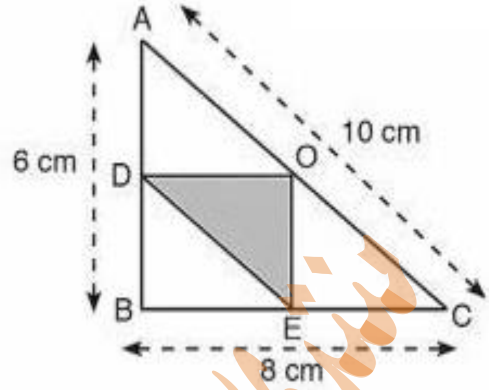
5) The image of the point (3 , -2) by translation (x , y) → (x - 1 , y + 6) is ....

(3) a) Prove that:

The ray drown parallel to one side of a triangle and passing through the midpoint of another side bisects the third side of the triangle.

b) In the opposite figure:

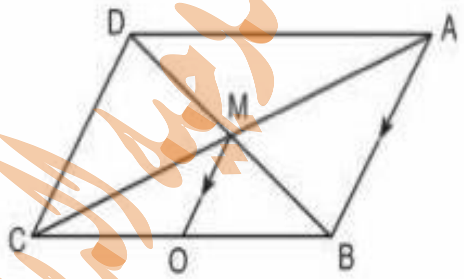
D , E , O are midpoint of $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ respectively, AB = 6 cm, BC = 8 cm, AC = 10 cm, find the perimeter of Δ DEO



(4) a) In the opposite figure:

ABCD is a parallelogram its diagonals are intersect at M , $\overrightarrow{MO}$ // $\overline{AB}$

Prove that: BO = OC

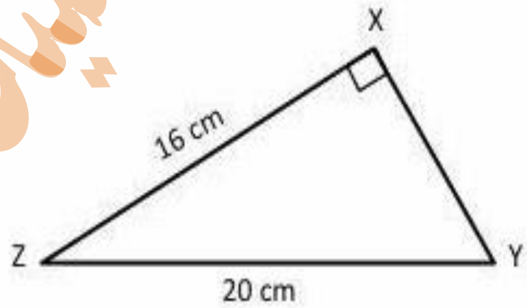


b) In the opposite figure:

XYZ is a triangle in which m ($\angle$ X) = 90°

YZ = 20 cm, XZ = 16 cm

Find the length of $\overline{XY}$

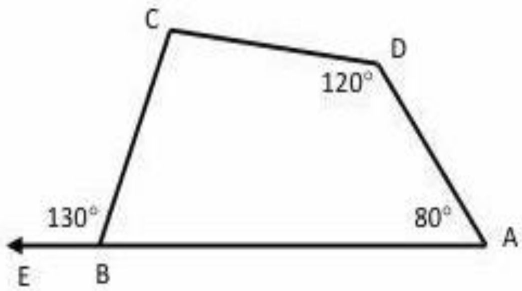


(5) a) In the opposite figure:

m ($\angle$ A) = 80° , m ($\angle$ D) = 120°,

m ($\angle$ CBE) = 130°

Find: m ($\angle$ C)



b) On a square lattice, draw the triangle whose vertices are A (4 , 4) , B (4 , 2) , C (1 , 2) then determine each of the following:

i) The coordinates of the image of Δ ABC by translation in direction $\overrightarrow{AB}$

ii) The image of Δ ABC by rotation about B and an angle of measure 180°

Model (6)

(1) Choose the correct answer:

1) The measure of each angle of the regular pentagon is ....

a) 90° b) 108° c) 120° d) 144°

2) The smallest number of the acute angle in any triangle is ....

a) zero b) 1 c) 2 d) 3

3) The rhombus of diagonals are equal in length is ....

a) square b) rectangle c) parallelogram d) trapezium

4) The image of the point (2 , -1) by reflection in x-axis is ....

a) (2 , 1) b) (1 , 2) c) (-2 , -1) d) (-1 , 2)

5) The image of the square by rotation about the origin point with an angle of measure 90° is ....

a) rectangle b) square c) rhombus d) trapezium

(2) Complete:

1) The measure of the exterior angle of a triangle is ....

2) The parallelogram whose diagonals are equal in length and perpendicular is ....

3) ABCD is a parallelogram in which m (∠ A) = 50° , then m (∠ B) = ...

4) The image of the point (-4 , 5) by translation (2 , -3) is ....

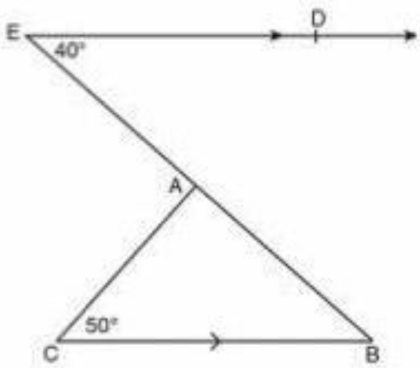
5) Δ XYZ, m (∠ Y) = 90° , the $(XZ)^2$ = ...

(3) a) In the opposite figure:

$\overrightarrow{ED} // \overline{CB}$, m (∠ C) = 50°

, m (∠ E) = 40°

Prove that: $\overline{AC} \perp \overline{BE}$



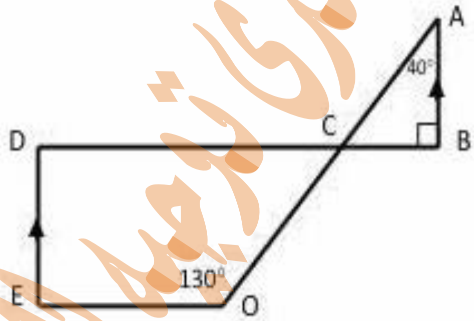
b) Draw Δ OBC on a square lattice where O (0 , 0) , B (3 , 0) , C (0 , 4) then find its image by rotation about the origin point with an angle of measure 180°.

(4) a) In the opposite figure:

$\overline{BD} \cap \overline{AO} = \{C\}$, $\overline{AB} \parallel \overline{DE}$, m ($\angle$ A) = 40°

, m ($\angle$ B) = 90° , m ($\angle$ COE) = 130°

Find: m ($\angle$ E)



b) In the opposite figure:

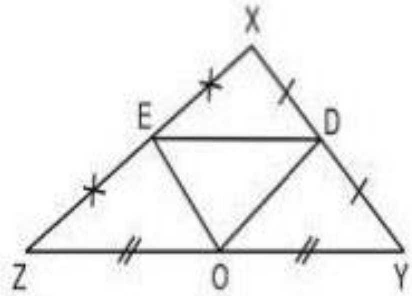
Find with proof the measures of the angles of Δ ABC



(5) a) In the opposite figure

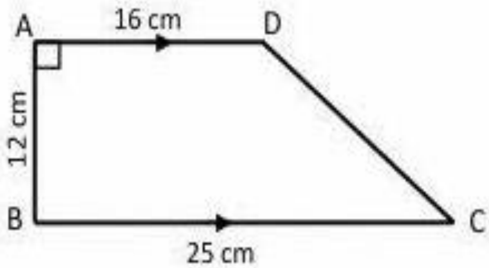
XYZ is a triangle in which XY = 6 cm , XZ = 8 cm , YZ = 12 cm ,

Find the perimeter of Δ DOE



b) In the opposite figure

ABC is a trapezium in which $\overline{AD} \parallel \overline{BC}$, m ($\angle$ A) = 90° , AB = 12 cm , BC = 25 cm, AD = 16 cm. Find the length of $\overline{DC}$



Model (7)

(1) Choose the correct answer:

1) The measure of each angle of the regular hexagon is ....

a) 108° b) 120° c) 136° d) 144°

2) Any triangle has at least two .... interior angles.

a) acute b) right c) obtuse d) straight

3) The diagonals of the rectangle are ....

a) parallel b) perpendicular

c) equal in the length d) equal in length & perpendicular

4) The image of a triangle by rotation about the origin point with an angle of measure 180° is ......

a) triangle b) line segment c) point d) straight line

5) The image of the point (3 , -2) by reflection in y-axis ......

a) (3 , 2) b) (-3 , -2) c) (-3 , 2) d) (-2 , 3)

(2) Complete:

1) The sum of the measures of the interior angles of a pentagon equals ....

2) The measure of the exterior angle of an equilateral triangle .... at one of its vertices.

3) The rectangle is a parallelogram in which one of its angles is ......

4) The image of the point (-4 , 5) by translation (2 , -3) is ....

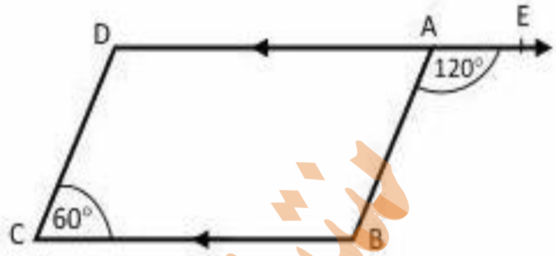
5) If the image of the point (-4 , 0) by rotation about the origin point is (0 , -4), then the measure of rotation angle is ....

(3) a) In the opposite figure:

E ∈ $\overrightarrow{DA}$, m (∠ EAB) = 120°

m (∠ C) = 60° , $\overrightarrow{DA}$ // $\overline{CB}$

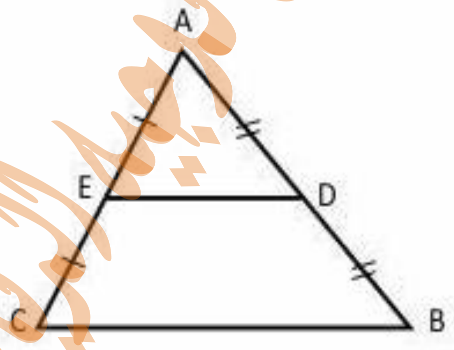
Prove that: ABCD is a parallelogram



b) In the opposite figure:

ABC is a triangle in which D, E are the midpoints of $\overline{AB}$, $\overline{AC}$ respectively,

DE = 5 cm. Find the length of $\overline{BC}$



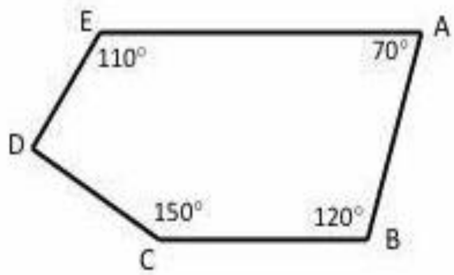
(4) a) In the opposite figure:

ABCD is a pentagon in which m (∠ A) = 70°

, m (∠ B) = 120°, m (∠ C) = 150°,

m (∠ E) = 110°

Find: m (∠ D) with proof



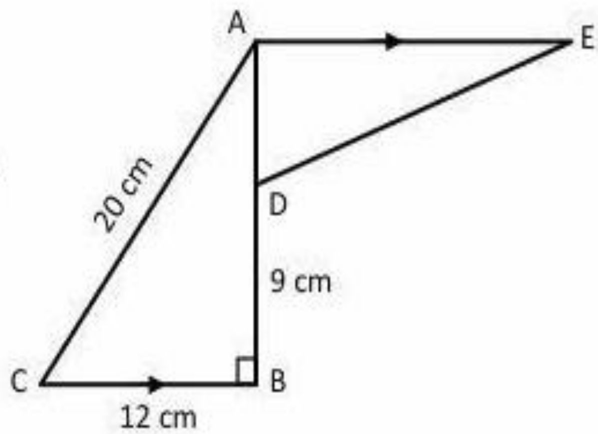
b) In the opposite figure:

Δ ABC , m (∠ B) = 90° , $\overline{AE}$ // $\overline{BC}$

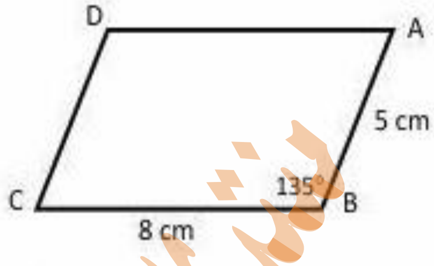
If BC = 12 cm , AC = 20 cm, D ∈ $\overrightarrow{AB}$

BD = 9 cm, AE = 2 BC

Find the length of each $\overline{AD}$, $\overline{ED}$



(5) a) In the opposite figure:



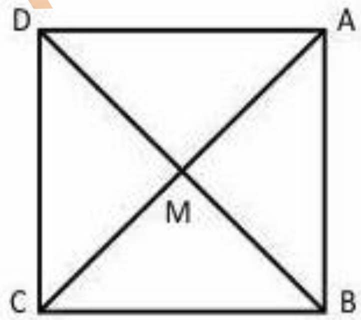
ABCD is a parallelogram in which

AB = 5 cm, BC = 8 cm, m (∠ B) = 135°

Find: i) m (∠ C)

ii) The perimeter of parallelogram ABCD

b) In the opposite figure:



ABCD is a square, whose diagonals intersect at M. Find:

First: The image of Δ ABC by reflection in $\overleftrightarrow{AC}$

Second: The image of Δ MAB by rotation about M with measure -90°

Model (8)

(1) Choose the correct answer:

1) The diagonal of square makes angle of measure ……….. with any of its sides.

a) 45° b) 60° c) 90° d) 120°

2) The measure of any of the exterior angle of an equilateral triangle equals ………. .

a) 60° b) 90° c) 120° d) 180°

3) In Δ ABC if m (∠ A) > (∠ B) + m (∠ C) then angle A is ……..

a) acute b) right c) obtuse d) straight

4) The sum of the measures of any two consecutive angles in a parallelogram equals …….

a) 90° b) 180° c) 270° d) 360°

5) The image of the point (5 , -3) by rotation about the origin point is itself, then the measure of rotation angle is ………..

a) 90° b) 180° c) 270° d) 360°

(2) Complete:

1) The length of a line segment joining the midpoints of two sides of a triangle equals ……………… .

2) The parallelogram whose diagonals are perpendicular and of equal in length is …………… .

3) The parallelogram whose perimeter 24 cm and the length of one of its sides is 7 cm, then the length of its adjacent side equals ……..

4) The image of the point (3 , 4) by reflection in the x-axis is …….. and its image by reflection in the y-axis is …………

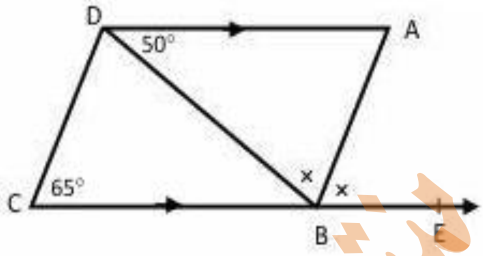
5) If the image of the point (-1 , 3) by a translation is (1 , 4) then the image of the point (3 , -2) by the same translation is ……….

(3) a) In the opposite figure:

$\overrightarrow{DA} \parallel \overrightarrow{BE}$, $\overrightarrow{BA}$ bisects $\angle$ DBE

, m ($\angle$ ADB) = 50°, m ($\angle$ C) = 65°

Prove that: ABCD is parallelogram.



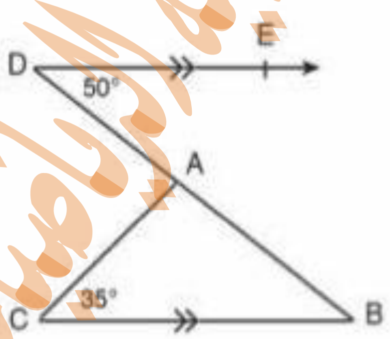
b) Draw the triangle ABC in which AB = AC = 5 cm, BC = 6 cm, then draw its image by rotation about A with an angle of measure 270°.

(4) a) In the opposite figure

$\overrightarrow{DE} \parallel \overrightarrow{CB}$, m ($\angle$ D) = 50°,

m ($\angle$ C) = 35°,

Find: m ($\angle$ B) , m ($\angle$ BAC)



b) Draw the triangle ABC in which AB = 3 cm, BC = 4 cm, m ($\angle$ ABC) = 90°, then draw its image by reflection in straight line $\overleftrightarrow{BC}$.

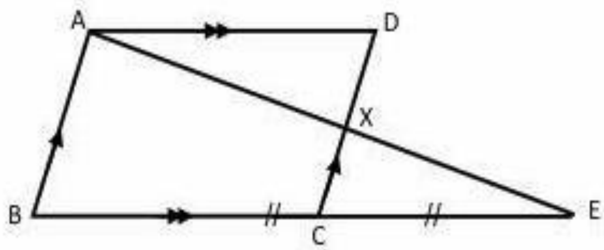
(5) a) In the opposite figure

ABCD is a parallelogram,

E ∈ $\overrightarrow{BC}$ Such that:

CE = BC , AE ∩ $\overline{DC}$ = {X}

Prove that: AX = XE

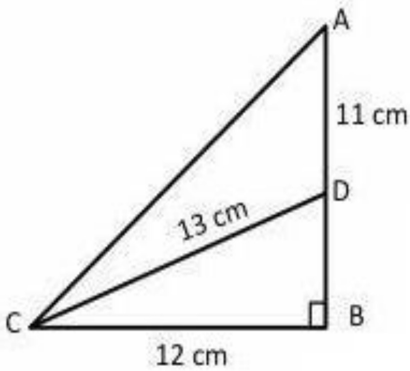


b) In the opposite figure

ABC is a triangle m ($\angle$ B) = 90°, D ∈ $\overline{AB}$,

AD = 11 cm if BC = 12 cm, DC = 13 cm

Find the length of $\overline{BD}$, $\overline{AC}$



Model Answer

Model (1)

(1) Choose the correct answer:

1) 180° 2) (3 , 1) 3) 120° 4) rhombus 5) 3

(2) Complete:

1) (2 , -1) 2) (-2 , 1) 3) all the sides are equal

4) 120° 5) (8 , 2)

(3) a)

m (∠ ACD) = 25 + 25 = 50°

b)

∵ D , E is midpoint of $\overline{AB}$, $\overline{AC}$

∴ ED = $\frac{1}{2}$ CB = 5 cm

∴ AD = 12 ÷ 2 = 6 cm

∴ AE = 8 ÷ 2 = 4 cm

∴ P. of Δ ADE = 5 + 6 + 4 = 15 cm

(4) a)

∵ $\overline{ED}$ // $\overline{ZY}$, $\overline{DZ}$ is transversal

∴ m (∠ Z) = m (∠ D) = 50° ← alternate angles

∵ Sum of interior angles of Δ = 180°

∴ m (∠ Y) = 180 - (50 + 105) = 25°

∵ (∠ DXY) is exterior angle of Δ XYZ

∴ m (∠ DXY) = 25 + 50 = 75°

b)

$\because \overline{AZ} \parallel \overline{YD} \parallel \overline{XE} \parallel \overline{CB}$,

AY = YX = XC

$\therefore$ AD = DE = EB = 18 ÷ 3 = 6 cm

(5) a)

In $\triangle$ ADC is right angled at D

$\therefore (CD)^2 = (AC)^2 - (AD)^2$

$= 9000 - 576 = 324$

$\therefore CD = \sqrt{324} = 18$ cm

In $\triangle$ ADB is right angled at D

$\therefore (DB)^2 = (AB)^2 - (AD)^2$

$= 676 - 576 = 100$

$\therefore DB = \sqrt{100} = 10$ cm

$\therefore$ CB = 10 + 18 = 28 cm

$\therefore$ A of $\triangle$ ABC $= \frac{1}{2} \times B \times H$

$= \frac{1}{2} \times 28 \times 24 = 336\ cm^2$

b)

$\because$ ABCD is a square

$\therefore \overline{AD} \parallel \overline{CB}$

$\because E \in \overrightarrow{BC}$

$\therefore \overline{AD} \parallel \overline{EC}$

$\because \overline{AD} \parallel \overline{EC}$, $\overline{AC} \parallel \overline{DE}$

$\therefore$ ACED is a parallelogram

Model (2)

(1) Choose the correct answer:

1) (2 , 5) 2) 120° 3) rectangle 4) c 4) $\frac{3}{8}$

(2) Complete:

1) 360° 2) (5 , 5) 3) 120°

4) bisect the third one 5) ZYC

(3) a) In △ XYZ is right angled at Y

$\therefore (XZ)^2 = (XY)^2 + (YZ)^2$

$= 49 + 576 = 625$

$\therefore XZ = \sqrt{625} = 25$ cm

In △ ZLX is right angled at L

$\therefore (LZ)^2 = (XZ)^2 - (LX)^2$

$= 625 - 225 = 400$

$\therefore LZ = \sqrt{400} = 20$ cm

b) A (4 , 3) $\xrightarrow{(x+2,\,y-1)}$ A` (6 , 2)

B (-1 , 1) $\xrightarrow{(x+2,\,y-1)}$ B` (1 , 0)

Draw by yourself

(4) a) A (1 , 1) $\xrightarrow{(x,\,-y)}$ A` (1 , -1)

B (3 , 4) $\xrightarrow{(x,\,-y)}$ B` (3 , -4)

C (5 , 2) $\xrightarrow{(x,\,-y)}$ C` (5 , -2)

Draw by yourself

b)

∵ sum of interior angles of △ ABC = 180°

∴ m (∠ C) = 180 - (30 + 90) = 60°

∴ m (∠ DCO) = m (∠ ACB) = 60° ← [V.O.A]

∵ sum of interior angles of any quadrilateral = 360°

∴ m (∠ E) = 360 - (120 + 60 + 90) = 90°

(5) a)

∵ $\overline{EO}$ // $\overline{CB}$, $\overline{B}$ is transversal

∴ m (∠ E) = m (∠ EBC) = 50° ← alternate angles

∵ sum of interior angles of △ ABC = 180°

∴ m (∠ CAB) = 180 - (30 + 50) = 100°

∵ (∠ ABD) is exterior angle of △ ABC

∴ m (∠ ABD) = 100 + 30 = 130°

b)

∵ $\overline{AD}$ // $\overline{XY}$ // $\overline{BC}$, AX = XB

∴ DY = YC

∵ Y is the midpoint of $\overline{CD}$, $\overline{ZY}$ // $\overline{ED}$

∴ CZ = ZE

Model (3)

(1) Choose the correct answer:

1) (-3 , -5) 2) 540° 3) 2 4) 5) 45°

(2) Complete:

1) all the sides are equal 2) equal 3) y

4) parallel to the third side 5) (-4 , -1)

(3) a)

A (1 , 0) $\xrightarrow{x-axis}$ A` (1 , 0)

B (0 , 2) $\xrightarrow{x-axis}$ B` (0 , -2)

C (-3 , 1) $\xrightarrow{x-axis}$ C` (-3 , -1)

Draw by yourself

b)

∵ ABCD is a parallelogram

∴ M is the midpoint of $\overline{AC}$

∵ M is the midpoint of $\overline{AC}$, $\overline{ME}$ // $\overline{CB}$

∴ E is the midpoint of $\overline{AB}$

∴ $\overline{AE} = \overline{EB}$

(4) a) ∵ $\overline{DE}$ // $\overline{CB}$, $\overline{CD}$ is transversal

∴ m (∠ C) = 180 - 120 = 60° ← interior on

∵ $\overline{OH}$ // $\overline{CB}$, $\overline{OB}$ is transversal

∴ m (∠ B) = 180 - 135 = 45° ← interior angles

∵ sum of interior angles of △ ABC = 180°

∴ m (∠ A) = 180 - (60 + 45) = 75°

b) In Δ ABC is right angled at B

$\therefore (AC)^2 = (AB)^2 + (CB)^2$

$= 144 + 81 = 225$ cm

$\therefore$ AC = 15 cm

In Δ ACD is right angled at C

$\therefore (AD)^2 = (AC)^2 + (CD)^2$

$= 225 + 400 = 625$

$\therefore$ AD = 25 cm

(5)

a) Δ LNM   b) Δ DOM   c) Δ BON

Model (4)

(1) Choose the correct answer:

1) rhombus   2) 108°   3) acute   4) 28 cm   5) (2 , 1)

(2) Complete:

1) parallel to the third side   2) equal in measure

3) each two opposite sides are parallel

4) (2 , -3)   5) (-3 , -2)

(3) $\because$ sum of interior angles of Δ EOC = 180°

$\therefore$ m (∠ EOC) = 180 - (35 + 50) = 95°

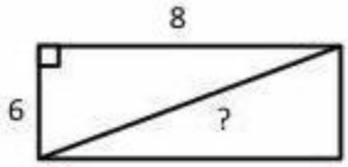
$\therefore$ m (∠ DOB) = m (∠ EOC) = 95°   ← [V.O.A]

$\because$ sum of interior angles of quadrilateral DOBA = 360°

$\therefore$ m (∠ B) = 360 - (85 + 100 + 95) = 80°

(4) a) $A = L \times w$

$48 = L \times 6 \qquad \rightarrow L = \frac{48}{6} = 8$ cm



$\therefore (\text{diagonal})^2 = L^2 + w^2$

$= 64 + 36 = 100$

diagonal $= \sqrt{100} = 10$ cm

b) $\because \overline{DE} \parallel \overline{AB}$, $\overrightarrow{DB}$ transversal

$\therefore m(\angle B) = m(\angle D) = 60°$ ← alternate angles

$\because (\angle DOA)$ is exterior angle of $\triangle OAB$

$\therefore m(\angle A) = m(\angle DOA) - m(\angle B)$

$= 110 - 60 = 50°$

(5) a) The point A = (1 , 2)

The image by translation (-1 , 2) = (0 , 4)

b) In $\triangle ABC$

$\because$ D is the midpoint of $\overline{AB}$, $\overline{ED} \parallel \overline{BC}$

$\therefore$ E is the midpoint of $\overline{AC}$

, $\overline{DE} = \frac{1}{2} BC = 6$ cm $\qquad \rightarrow (1)$

$\because$ O is the midpoint of $\overline{BC}$

$\therefore OB = \frac{1}{2} BC = 6$ cm

$\because$ In quadrilateral EOBD

$\because \overline{ED} \parallel \overline{OB}$, ED = OB = 6 cm

$\therefore$ EOBD is parallelogram

$\because$ D , O is midpoint of $\overline{AB}$, $\overline{BC}$

$\therefore$ OD = $\frac{1}{2}$ AC = 4 cm → (2)

$\because$ E , O is midpoint of $\overline{AC}$, $\overline{BC}$

$\therefore$ EO = $\frac{1}{2}$ AB = 5 cm → (3)

From (1) , (2) , (3)

$\therefore$ P. of Δ EOD = 6 + 4 + 5 = 15 cm

Model (5)

(1) Choose the correct answer:

1) b 2) d 3) a 4) d 5) c

(2) Complete:

1) rhombus 2) right-angled 3) acute

4) (2 , 4) 5) (2 , 4)

(3) a) Look at p. 66 in pupils book

b) $\because$ D is the midpoint of $\overline{AB}$, O is the midpoint of $\overline{AC}$

$\therefore$ DO = $\frac{1}{2}$ BC $\therefore$ DO = 4 cm

$\because$ O is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

$\therefore$ OE = $\frac{1}{2}$ AB $\therefore$ OE = 3 cm

$\because$ E is the midpoint of $\overline{BC}$, D is the midpoint of $\overline{AB}$

$\therefore$ ED = $\frac{1}{2}$ AC $\therefore$ ED = 5 cm

$\therefore$ The perimeter of Δ DEO = 4 + 3 + 5 = 12 cm

(4) a) ∵ ABCD is a parallelogram ,

M is the intersection point of its diagonals

∴ M is the midpoint of $\overline{AC}$

In the Δ ABC:

∵ $\overrightarrow{MO}$ // $\overline{AB}$, M is the point of $\overline{AC}$

∴ O is the midpoint of $\overline{BC}$

∴ BO = OC

b) In the Δ XYZ:

∵ m ($\angle$ X) = 90°

∴ $(XY)^2 = (YZ)^2 - (XZ)^2 = 400 - 256 = 144$

∴ $XY = \sqrt{144} = 12$ cm

(5) a)

∵ B ∈ $\overrightarrow{AE}$

∴ ($\angle$ ABC) = 180 - 130 = 50°

∵ The sum of measure of interior angles of quadrilateral = 360°

∴ m ($\angle$ C) = 360 - (50 + 80 + 120°)

= 360 - 250 = 110°

b) AB = | B - A | = |2 - 4| = |- 2| = 2 units

2 AB = 2 × 2 = 4 units

The direction of $\overrightarrow{AB}$ is in the positive direction of y-axis

Model (6)

(1) Choose the correct answer:

1) a 2) c 3) a 4) a 5) b

(2) Complete:

1) equal to the sum of the measures of its non-adjacent interior angles.

2) square 3) 130° 4) (-2 , 2) 5) $(XY)^2 + (YZ)^2$

(3) a) $\because \overrightarrow{ED} \parallel \overline{CB}$, $\overrightarrow{BE}$ is a transversal

$\therefore$ m (∠ B) = m (∠ E) = 40° (alternate angles)

$\because$ The sum of measures of interior angles of triangle = 180°

$\therefore$ m (∠ BAC) = 180 - (50 + 40) = 90°

$\therefore \overline{AC} \perp \overline{BE}$

b) O (0 , 0) $\xrightarrow[(0,\,180^\circ)]{\text{ROT.}}$ O` (0 , 0)

B (3 , 0) $\xrightarrow[(0,\,180^\circ)]{\text{ROT.}}$ B` (- 3 , 0)

C (0 , 4) $\xrightarrow[(0,\,180^\circ)]{\text{ROT.}}$ C` (0 , - 4) Draw by yourself.

(4) a) $\because$ The sum of measures of interior angles of Δ = 180°

$\therefore$ m (∠ ACB) = 180 - (90 + 40) = 50°

$\because \overline{BD} \cap \overline{AO} = \{C\}$

$\therefore$ m (∠ DCO) = m (∠ ACB) = 50° (V.O.A)

$\because \overline{AB} \parallel \overline{DE}$

$\therefore$ m (∠ B) = m (∠ D) = 90° (alternate angles)

$\because$ The sum of measures of interior angles of quadrilateral = 360°

$\therefore$ m (∠ E) = 360 - (90 + 50 + 130) = 90°

b) $\because$ The sum of measures of interior angles of $\Delta = 180°$

$\therefore x + 3x + 2x = 180°$

$6x = 180°$

$x = \frac{180}{6} = 30°$

$\therefore m(\angle A) = x = 30°$, $m(\angle B) = 3x = 90°$

$m(\angle C) = 2x = 60°$

(5) a) In Δ XYZ

$\because$ D is the midpoint of $\overline{XY}$, E is the midpoint of $\overline{XZ}$

$DE = \frac{1}{2} ZY = \frac{1}{2} \times 12 = 6$ cm

$\because$ E , O are midpoint of $\overline{XZ}$ and $\overline{ZY}$

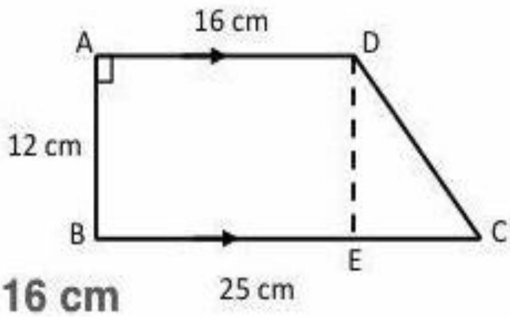
$\therefore EO = \frac{1}{2} \times XY = \frac{1}{2} \times 6 = 3$ cm

$\because$ O , D are midpoints of $\overline{XY}$

$\therefore OD = \frac{1}{2} \times XZ = \frac{1}{2} \times 8 = 4$ cm

$\therefore$ The p. of Δ DOE $= 6 + 3 + 4 = 13$ cm

b)



Construction: Draw $\overline{DE} \perp \overline{BC}$

Proof: $\because$ ABED is a rectangle

$\therefore DE = AB = 12$ cm , $AD = BE = 16$ cm

$\therefore EC = 25 - 16 = 9$ cm

$\because m(\angle DEC) = 90°$

$\therefore (DC)^2 = (DE)^2 + (EC)^2 = 144 + 81 = 225$

$DC = \sqrt{225} = 15$ cm

Model (7)

(1) Choose the correct answer:

1) b 2) a 3) c 4) a 5) b

(2) Complete:

1) 540° 2) 120° 3) right 4) (-2 , 2) 5) 90°

(3) a) $\because \overrightarrow{DA} \mathbin{/\mkern-6mu/} \overline{BC}$

$\therefore$ m ($\angle$ B) = m ($\angle$ EAB) = 120° (alternate angles)

$\because$ m ($\angle$ B) = m ($\angle$ C) = 120 + 60 = 180°

and they are interior angles

$\therefore \overline{AB} \mathbin{/\mkern-6mu/} \overline{CD}$

$\because \overline{AB} \mathbin{/\mkern-6mu/} \overline{CD}$ & $\overline{AD} \mathbin{/\mkern-6mu/} \overrightarrow{CB}$

$\therefore$ ABCD is a parallelogram

b) In the Δ ABC:

$\because$ D, E are the midpoints of $\overline{AB}, \overline{AC}$

$\therefore ED = \frac{1}{2} BC$ BC = 2 ED

$\therefore BC = 2 \times 5 = 10$ cm

(4) a) $\because$ The sum of measures of interior angles of the pentagon

$= (5 - 2) \times 180° = 540°$

$\therefore$ m ($\angle$ D) = 540 - (110 + 70 + 120 + 150) = 90°

b) In Δ ABC: $\because$ m ($\angle$ B) = 90°

$\because (AB)^2 = (AC)^2 - (BC)^2 = 400 - 144 = 256$

$\therefore$ AB = 16 cm

$\therefore$ AD = AB - DB = 16 - 9 = 7 cm

$\because$ AE = 2 BC $\quad\quad\quad$ $\therefore$ AE = 2 × 12 = 24

, $\because \overline{AE} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m ($\angle$ A) = m ($\angle$ B) = 90° (alternate angle)

$\therefore (ED)^2 = (AD)^2 + (AE)^2 = 49 + 576 = 625$

$\therefore$ ED = $\sqrt{625}$ = 25

(5) a) $\because$ ABCD is a parallelogram

$\therefore$ m ($\angle$ C) = 180° - m ($\angle$ B) = 180 - 135 = 45°

$\because$ ABCD is a parallelogram

$\therefore$ AB = CD = 5 cm , AD = BC = 8 cm

P = 5 + 8 + 5 + 8 = 26 cm

b) First: $\triangle$ ADC

Second: $\triangle$ MBC

Model (8)

(1) Choose the correct answer:

1) a $\quad$ 2) c $\quad$ 3) c $\quad$ 4) b $\quad$ 5) d

(2) Complete:

1) half the length of the third side

2) square $\quad\quad$ 3) 5 cm

4) (3 , -4) , (-3 , 4)

5) translation = (2 , 10)

image = (5 , -1)

(3) a) $\because \overrightarrow{DA} \parallel \overrightarrow{BE}$

$\therefore$ m ($\angle$ CBD) = m ($\angle$ ADB) = 50° (alternate angles)

$\therefore$ m ($\angle$ DBE) = 180 - 50 = 130°

$\because \overrightarrow{BA}$ bisects $\angle$ DBE

m ($\angle$ ABE) = $\frac{130°}{2}$ = 65°

, $\because$ m ($\angle$ C) = m ($\angle$ ABE) = 65°

and they are corresponding angles

$\therefore \overline{AB} \parallel \overline{CD}$

$\because \overline{AD} \parallel \overline{CB}$, $\overline{AB} \parallel \overline{CD}$

$\therefore$ ABCD is a parallelogram

b) Draw by yourself

(4) a)

$\because \overrightarrow{DE} \parallel \overline{CB}$, $\overleftrightarrow{BD}$ is a transversal

$\therefore$ m ($\angle$ B) = m ($\angle$ D) 50° (alternate angles)

In the Δ ABC:

$\because$ The sum of measures of interior angles of the triangle = 180°

$\therefore$ m ($\angle$ BAC) = 180 - (35 + 50) = 95°

b) Draw by yourself

(5) a) $\because$ ABCD is a parallelogram,

$\therefore \overline{AB} \parallel \overline{DC}$ $\therefore \overline{XC} \parallel \overline{AB}$

In Δ ABE: $\because$ C is midpoint of $\overline{BE}$, $\overline{XC} \parallel \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AE}$

$\therefore$ AX = XE

b) In $\triangle$ BCD:

$\because m(\angle B) = 90°$

$\therefore (BD)^2 = (DC)^2 - (BC)^2$

$= 169 - 144 = 25$

$BD = \sqrt{25} = 5$ cm

$AB = 5 + 11 = 16$ cm

In $\triangle$ ABC:

$\because (\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 256 + 144 = 400$

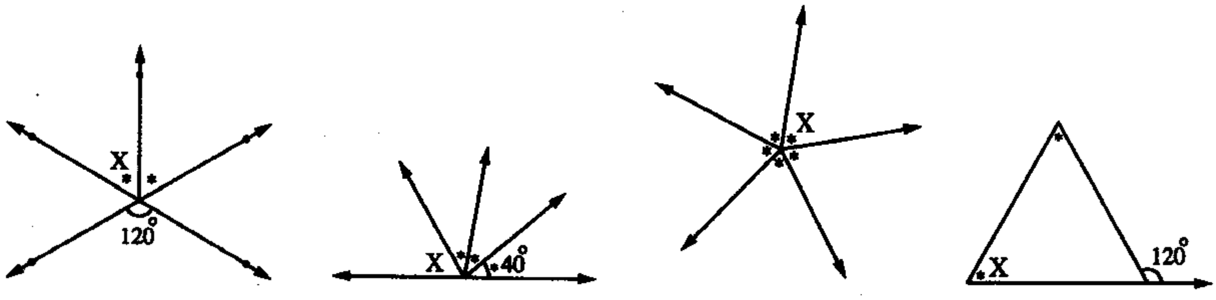
$AC = \sqrt{400} = 20$ cm

SECOND: GEOMETRY

Choose the correct answer :

1.	The sum of the measures of the interior angles of a triangle equals .... (a) 90° (b) 180° (c) 270° (d) 360°
2.	The image of the point (– 1 , 3) by translation (4 , – 2) is .... (a) (3 , 1) (b) (3 , – 1) (c) (5 , 1) (d) (5 , – 5)
3.	The measure of the exterior angle of the equilateral triangle is .... (a) 30° (b) 45° (c) 60° (d) 120°
4.	In a parallelogram if the adjacent sides are equal in the length , then the shape is .... (a) square. (b) rhombus. (c) rectangle. (d) trapezium.
5.	The number of the diagonals of a pentagon is .... (a) 3 (b) 5 (c) 7 (d) 9
6.	The image of the point (2 , – 5) by reflection in x-axis is .... (a) (2 , – 5) (b) (2 , 5) (c) (– 2 , – 5) (d) (5 , 2)
7.	The measure of each angle of regular hexagon equals .... (a) 60° (b) 108° (c) 120° (d) 135°
8.	The two diagonals are equal in the length and not perpendicular in .... (a) parallelogram. (b) rectangle. (c) rhombus. (d) square.
9.	The sum of the measures of the interior angles of a triangle = ...° (a) 90 (b) 360 (c) 180 (d) 540

10. In all the following shapes m (∠ X) = 60° except the shape ....

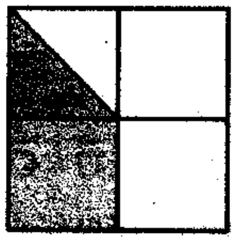


(a) (b) (c) (d)

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11. **In the opposite figure :**

The area of the shaded part from the area of all shape equals ....



(a) $\frac{1}{8}$ **(b) $\frac{1}{4}$** **(c) $\frac{3}{8}$** **(d) $\frac{3}{4}$**

---

12. The image of the point (3 , – 2) by reflection in the y-axis is the point ....

(a) (3 , 2) (b) (– 3 , – 2) (c) (– 3 , 2) (d) (– 2 , 3)

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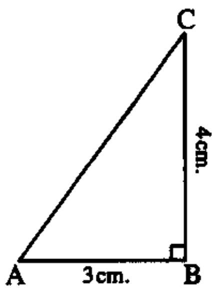
13. The diagonals are equal and perpendicular in ....

(a) rhombus. (b) square. (c) recangle. (d) parallelogram.

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14. **In the opposite figure :**

AC = ... cm.



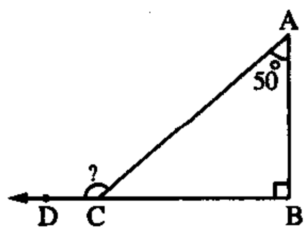
(a) 5 (b) 7

(c) 25 (d) 625

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15. **In the opposite figure :**

m (∠ ACD) = ...°



(a) 40 (b) 140

(c) 90 (d) 50

16.	The reflected image of the point A (– 3 , 2) in the origin point is the point À (............... , ...) (a) (3 , – 2) (b) (3 , 2) (c) (– 3 , – 2) (d) (2 , – 3)
17.	The reflection in the X-axis maps the point B (X , y) to the point B̀ (............... , ...) (a) (X , y) (b) (X , – y) (c) (– X , – y) (d) (– X , y)
18.	✱ The measure of the exterior angle of the equilateral triangle at any one of its vertices equals .... (a) 60° (b) 120° (c) 150° (d) 30°
19.	The image of the point (– 1 , 3) under the translation (4 , – 2) is the point (............... , ...) (a) (5 , – 5) (b) (5 , 1) (c) (3 , 1) (d) (3 , – 1)
20.	✱ The measure of the interior angle of a regular polygon of 10 sides equals .... (a) 72° (b) 108° (c) 144° (d) 150°
21.	The image of the point (– 4 , 5) by translation (2 , – 3) is .... (a) (2 , 2) (b) (– 2 , 2) (c) (2 , – 2) (d) (– 2 , – 2)
22.	The point whose image by reflection in the origin point is itself is .... (a) (1 , 0) (b) (0 , 1) (c) (0 , 0) (d) (– 1 , 0)
23.	The image of a triangle by rotation about the origin point with an angle of measure 360° is .... (a) point. (b) triangle. (c) line segment. (d) straight line.
24.	If ABC is right-angled triangle at B , AB = 20 cm. , AC = 25 cm. , then the length of BC = ... cm. (a) 5 (b) 45 (c) 225 (d) 15
25.	The number of axis of symmetry of a square equal .... (a) 0 (b) 1 (c) 2 (d) 4

26. ✱ ABCD is a parallelogram in which : m (∠ A) = 50° , then m (∠ B) = ……………
(a) 50° (b) 120° (c) 130° (d) 40°

27. The line segment joining the midpoints of two sides of a triangle is …………… the third side.
(a) bisect to (b) perpendicular (c) equal to (d) parallel to

28. The image of a rhombus by any translation is a ……………
(a) rhombus. (b) rectangle. (c) square. (d) trapezium.

29. ABC is a triangle in which m (∠ A) = 90° , then $(AC)^2 = (BC)^2$ …………… $(AB)^2$
(a) + (b) − (c) × (d) ÷

30. The image of the point (− 1 , 3) by reflection in y-axis is ……………
(a) (1 , 3) (b) (3 , − 1) (c) (− 1 , − 3) (d) (1 , − 3)

31. ✱ In Δ ABC : if m (∠ A) + m (∠ B) = 110° , then m (∠ C) = ……………
(a) 110° (b) 90° (c) 70° (d) 55°

32. The number of diagonals of a quadrilateral is ……………
(a) 4 (b) 3 (c) 2 (d) 0

33. The diagonal of square divided its vertex angle in two angles of the measure of each of them is ……………
(a) 30° (b) 45° (c) 60° (d) 90°

34. The image of the point (− 3 , 5) by rotation about the origin point and with an angle of measure 90° is ……………
(a) (5 , 3) (b) (− 5 , 3) (c) (3 , 5) (d) (− 5 , − 3)

35. Any triangle has at least two …………… interior angles.
(a) right (b) obtuse (c) acute (d) reflex

36. The sum of measures of the interior angles of the pentagon is ……………
(a) 540° (b) 720° (c) 360° (d) 180°

37. The image of the point (− 2 , 3) by reflection in y-axis is ……………
(a) (2 , 3) (b) (− 2 , − 3) (c) (2 , − 3) (d) (3 , 2)

38.	The image of the point $(-3, 5)$ by rotation about the origin point with angle of measure $90°$ is …………… (a) $(5, 3)$ (b) $(-5, -3)$ (c) $(3, 5)$ (d) $(-5, 3)$
39.	In Δ ABC if m ($\angle$ B) = $90°$, AB = 6 cm., BC = 8 cm., then AC = …………… cm. (a) 100 (b) 8 (c) 6 (d) 10
40.	The image of the point $(2, -3)$ by translation $(4, 1)$ is …………… (a) $(6, 2)$ (b) $(4, -2)$ (c) $(6, -2)$ (d) $(-2, 6)$

Complete each of the following :

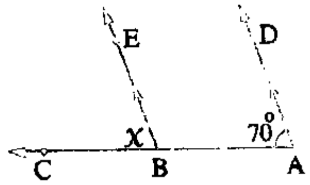
1.	The image of the point $(2, 1)$ by reflection in x-axis is ……………
2.	The image of the point $(2, -1)$ by rotation about the origin point with an angle of measure $180°$ is ……………
3.	The square is a rectangle in which ……………
4.	ABCD is a parallelogram in which m ($\angle$ A) = $60°$, then m ($\angle$ B) = ……………
5.	The image of the point $(5, 3)$ by translation : $(x, y) \longrightarrow (x + 3, y - 1)$ is ……………
6.	The sum of the measures of the angles of the quadrilateral equals ……………
7.	The image of the point $(2, 3)$ by translation MN, in direction $\overrightarrow{MN}$, where M $(2, -1)$, N $(5, 1)$ is ……………
8.	ABCD is a parallelogram in which m ($\angle$ A) = $60°$, then m ($\angle$ B) = ……………
9.	The ray drawn parallel to one side of a triangle and passing through the midpoint of another side ……………
10.	The length of the line segment that joins two midpoints of two sides of a triangle equals …………… the length of the third side.
11.	The rectangle is a parallelogram in which one of it's angles is ……………

12. The length of the side of a rhombus whose perimeter is 24 cm. equals .... cm.

13. The image of the point A $(-3\ ,2)$ by reflection in the origin point is the point À (............... , ...)

14. **In the opposite figure :**

x = ...°



15. The image of the point A $(4\ ,3)$ by rotation around the origin point O and an angle whose measure is 360° is the point À (............... , ...)

16. If À $(-3\ ,-3)$ is the reflected image of the point A $(x\ ,y)$ in the origin point $(0\ ,0)$, then x = ... and y = ...

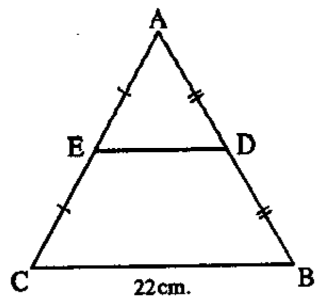
17. The image of the point C $(x\ ,y)$ is the point C̀ $(-x\ ,-y)$ by rotation around the origin point O and an angle whose measure is ....°

18. ✱ If ABCD is a parallelogram in which $m(\angle A) + m(\angle C) = 110°$, then $m(\angle D)$ = ...°

19. **In the opposite figure :**

If BC = 22 cm.

, then ED = ... cm.



20. Rotation in the plane reserves .... of angles.

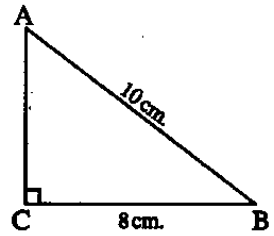
21. The line segment joining the midpoints of two sides of a triangle is .... the third side.

22. In the right-angled triangle , area of the square drawn on the hypotenuse equals .... of the lengths of the other two sides.

23. The sum of measures of the exterior angles of the hexagon equals .... °

24. The image of the point $(3\ ,-8)$ by rotation about the origin point with an angles of measure 180° is ....

25. In the rectangle ABCD , $(AB)^2 + (AD)^2$ = ...

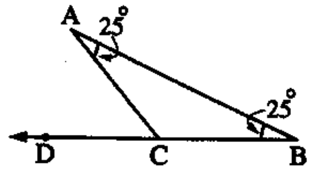
26.	The square is a …………… with a right angle.
27.	The length of the line segment joining the midpoints of two sides of a triangle equals …………… **In the opposite figure :** AC = …………… cm. 
28.	The image of the point (2 , 1) by reflection x-axis is ……………
29.	The image of the point (2 , 5) by translation $(x , y) \longrightarrow (x + 2 , y - 3)$ is ……………
30.	The measure of each interior angle of the regular pentagon is ……………
31.	A rhombus with right-angle is called ……………
32.	In the parallelogram XYZL , if m (∠ X) = $\frac{1}{2}$ m (∠ Y) , then m (∠ Y) = ……………°
33.	The line segment joining between the midpoints of two sides of triangle is ……………
34.	The image of (3 , 7) by rotation R (O , 180°) is ……………
35.	If À is the image of A by reflection in M and MA = 6 cm. , then AÀ = …………… cm.
36.	If ABCD is a rhombus , then …………… ⊥ ……………
37.	The image of the point (– 1 , 2) by rotation about the origin point with angle of measure 90° is ……………
38.	The sum of the measures of the accumulative angles about a point = ……………
39.	If Δ XYZ is a right angled triangle at X , XY = 12 cm. and XZ = 9 cm. , then YZ = ……………
40.	The line segment joinig the midpoints of two sides of a triangle is …………… the third side.

Essay problems:

1. **In the opposite figure :**

$m(\angle A) = m(\angle B) = 25°$

Find : $m(\angle ACD)$

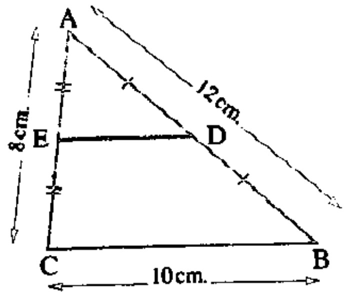


2. **In the opposite figure :**

ΔABC in which : AB = 12 cm. ,

BC = 10 cm. , AC = 8 cm.

Find : The perimeter of ΔADE

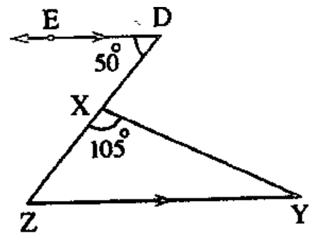


3. **In the opposite figure :**

$\overrightarrow{DE} \parallel \overline{YZ}$, $m(\angle ZDE) = 50°$

, $m(\angle YXZ) = 105°$

Find : $m(\angle Z)$, $m(\angle Y)$, $m(\angle YXD)$

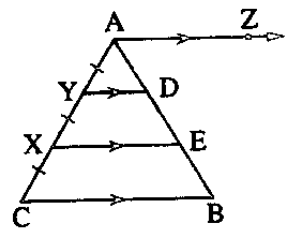


4. **In the opposite figure :**

$\overrightarrow{AZ} \parallel \overline{YD} \parallel \overline{XE} \parallel \overline{CB}$,

AY = YX = XC , AB = 18 cm. ,

Find : The length of $\overline{EB}$



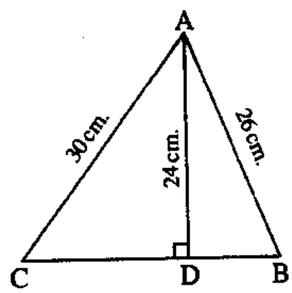
5. **In the opposite figure :**

$\overline{AD} \perp \overline{BC}$, if AD = 24 cm. ,

AB = 26 cm. , AC = 30 cm.

(1) **Find :** The length of $\overline{BC}$

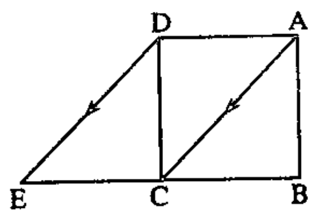
(2) **Find :** The area of ΔABC



6. **In the opposite figure :**

ABCD is a square , $E \in \overrightarrow{BC}$, $\overline{AC} \parallel \overline{DE}$

Prove that : ACED is a parallelogram.



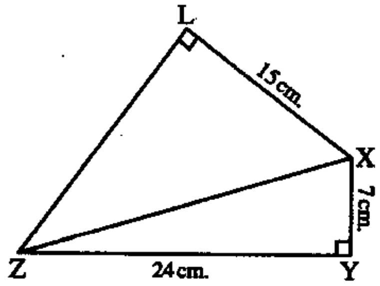
7. **In the opposite figure :**

XYZL is a quadrilateral in which

m (∠ Y) = m (∠ L) = 90° , XY = 7 cm. ,

YZ = 24 cm. , XL = 15 cm.

Find : The length of each of $\overline{XZ}$ and $\overline{LZ}$



8. **In the opposite figure :**

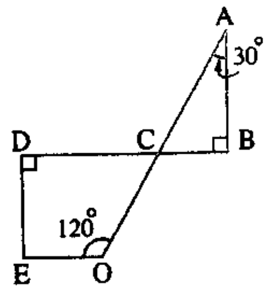
$\overline{AB}$ and $\overline{ED}$ are perpendicular to

$\overline{BD}$, $\overline{BD} \cap \overline{AO} = \{C\}$,

m (∠ A) = 30°

, m (∠ EOC) = 120° ,

Find : m (∠ E)

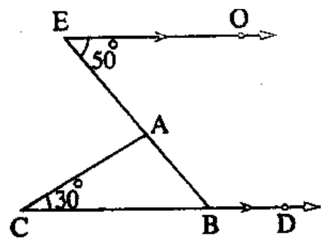


9. **In the opposite figure :**

$\overrightarrow{EO} // \overline{CD}$, m (∠ E) = 50°

, m (∠ C) = 30° ,

Find the measures of angles of Δ ABC , m (∠ ABD)



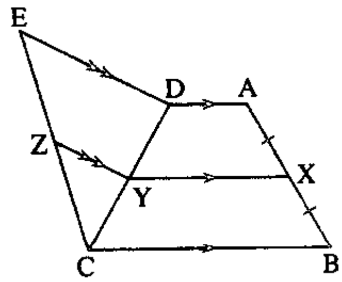
10. **In the opposite figure :**

X is the midpoint of $\overline{AB}$

, Y ∈ $\overline{CD}$, Z ∈ $\overline{CE}$

, $\overline{AD} // \overline{XY} // \overline{BC}$, $\overline{YZ} // \overline{DE}$

Is CZ = ZE ? giving reason



11. Find $\overline{A'B'}$ the translated image of $\overline{AB}$, where A (2 , 1) and B (2 , 4) when translated MN units in the direction of $\overrightarrow{MN}$, where M (– 2 , 5) and N (3 , 7)

12. **In the opposite figure :**

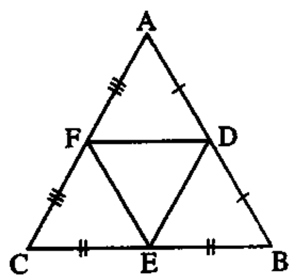
AB = 10 cm. , BC = 16 cm. , and AC = 14 cm. ,

D , E and F are the midpoints of

$\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ respectively.

Prove that :

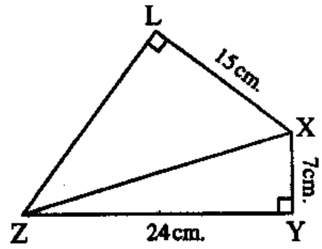
The perimeter of Δ DEF = 20 cm.



13.

In the opposite figure :

Find the length of $\overline{LZ}$



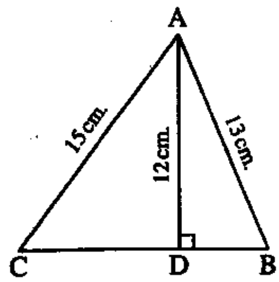
14.

In the opposite figure :

Prove that :

① BC = 14 cm.

② Area of Δ ABC = 84 cm^2.



15.

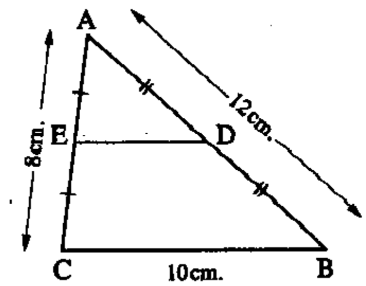
In the opposite figure :

ABC is a triangle in which

AB = 12 cm. , BC = 10 cm.

and AC = 8 cm.

Find the perimeter of the triangle ADE



16.

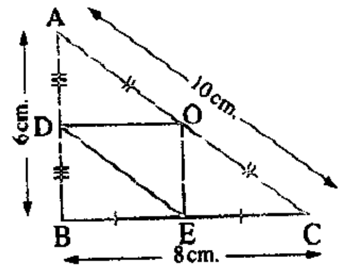
In the opposite figure :

ABC is a triangle in which D , E , O are midpoints

of $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ resectively , $\overline{DE}$ // $\overline{AC}$

AB = 6 cm. , BC = 8 cm. , AC = 10 cm.

Find with prove the perimeter of : Δ EDO



17.

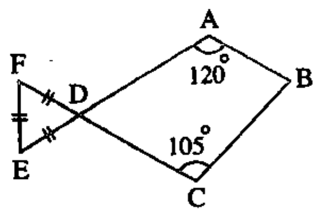
✱ In the opposite figure :

$\overline{AE} \cap \overline{CF} = \{D\}$

, Δ DEF is an equilateral triangle

, m ($\angle$ A) = 120° and m ($\angle$ C) = 105°

Find : m ($\angle$ B)



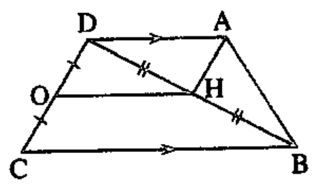
18.

In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, AD = $\frac{1}{2}$ BC

, H and O are midpoints of $\overline{DB}$ and $\overline{DC}$ respectively

Prove that : AHOD is a parallelogram



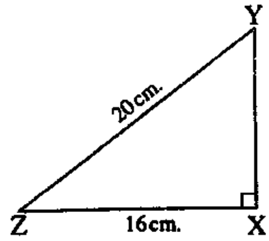
19.

In the opposite figure :

XYZ is right-angled triangle at X

, YZ = 20 cm. , XZ = 16 cm.

Find : the length of $\overline{XY}$



20.

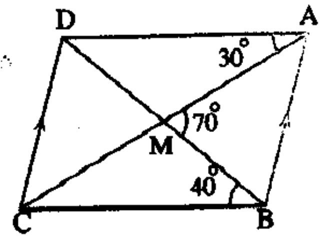
✱ In the opposite figure :

ABCD is a quadrilateral where : $\overline{AC} \cap \overline{BD} = \{M\}$

, $\overline{AB} \parallel \overline{DC}$, m (∠ AMB) = 70° , m (∠ MBC) = 40°

and m (∠ MAD) = 30°

Prove that : ABCD is a parallelogram.



21.

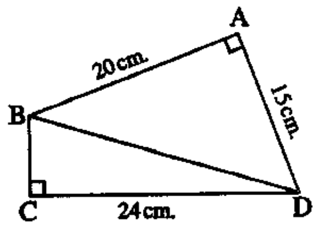
In the opposite figure :

m (∠ A) = m (∠ C) = 90°

, AB = 20 cm. , CD = 24 cm.

, AD = 15 cm.

Find : the perimeter of Δ BCD



22.

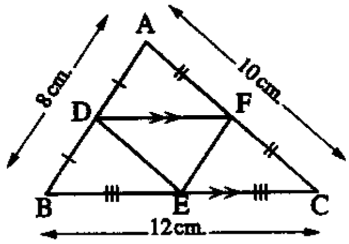
In the opposite figure :

D , E , F are midpoints of

$\overline{AB}$, $\overline{BC}$, $\overline{CA}$ respectively

, AB = 8 cm. , BC = 12 cm. , AC = 10 cm.

Find : the perimeter of Δ DEF



23.

In the opposite figure :

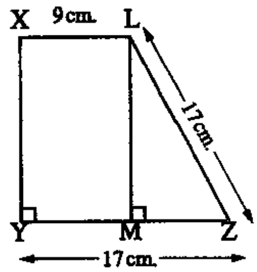
XYZL is a trapezium , in which

$\overline{XL} \parallel \overline{YZ}$, m (∠ Y) = 90°

, $\overline{LM} \perp \overline{YZ}$, YZ = ZL = 17 cm. , XL = 9 cm.

Find : ① The length of $\overline{XY}$

② The image of $\overline{XL}$ by translation of magnitude XY in the direction of $\overrightarrow{XY}$



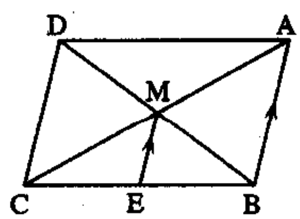
24.

In the opposite figure :

ABCD is a parallelogram its

diagonals are intersect at M

, $\overrightarrow{ME} \parallel \overline{AB}$ prove that BE = EC



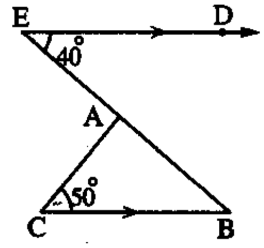
25. Find the length of the diagonal of a rectangle whose area 48 cm^2. and of width 6 cm.

26. **In the opposite figure :**

$\overrightarrow{ED} // \overline{CB}$, m (∠ C) = 50°

, m (∠ E) = 40°

Prove that : $\overline{AC} \perp \overline{BE}$

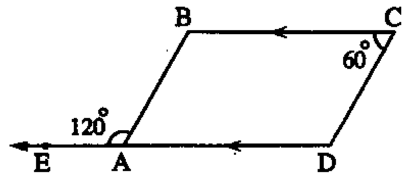


27. **In the opposite figure :**

E ∈ $\overrightarrow{DA}$, m (∠ EAB) = 120°

, m (∠ C) = 60° , $\overrightarrow{DA} // \overline{CB}$

Prove that : ABCD is a parallelogram

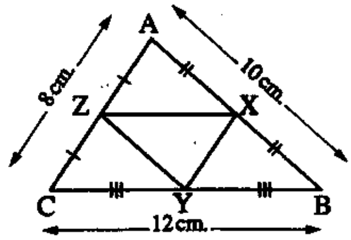


28. **In the opposite figure :**

ABC is a triangle in which X , Y , Z are midpoints of

$\overline{AB}$, $\overline{BC}$ and $\overline{CA}$, AB = 10 cm. , BC = 12 cm. , AC = 8 cm.

Find with proof : perimeter of Δ XYZ

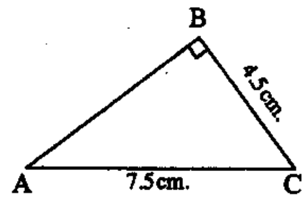


29. **In the opposite figure :**

ABC is a right-angled triangle at B

, AC = 7.5 cm. , BC = 4.5 cm.

Find : the length of $\overline{AB}$



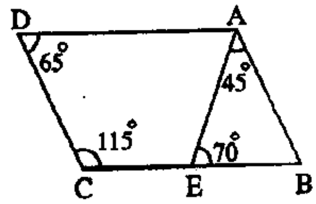
30. **✱ In the opposite figure :**

E ∈ $\overline{BC}$, m (∠ BAE) = 45°

, m (∠ AEB) = 70° , m (∠ D) = 65°

and m (∠ C) = 115°

Prove that : ABCD is a parallelogram.

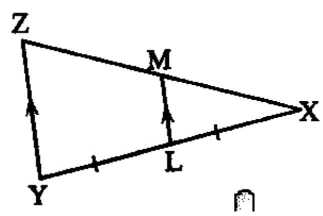


31. **In the opposite figure :**

L is a midpoint of $\overline{XY}$

, $\overline{LM} // \overline{YZ}$, XZ = 10 cm.

Find : the length of $\overline{XM}$

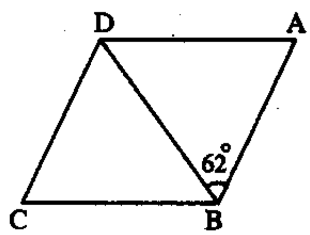


32. **✱ In the opposite figure :**

ABCD is a rhombus , $\overline{BD}$ is a diagonal in it

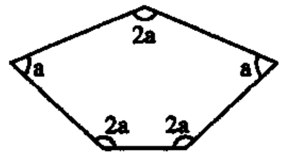
, m (∠ ABD) = 62°

Find with proof : m (∠ A)



33. **✱ In the opposite figure :**

Find the value of a



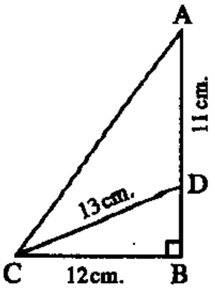
34. **In the opposite figure :**

Δ ABC in which m ($\angle$ B) = 90°

D $\in \overline{AB}$ such that AD = 11 cm.

, if BC = 12 cm , DC = 13 cm.

Find the length of each of : $\overline{AB}$, $\overline{AC}$

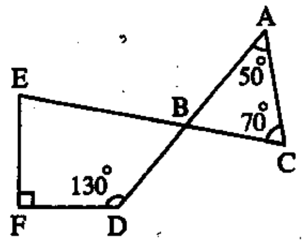


35. **✱ In the opposite figure :**

$\overline{CE} \cap \overline{AD} = \{B\}$, m ($\angle$ A) = 50° , m ($\angle$ C) = 70°

, m ($\angle$ D) = 130° and m ($\angle$ F) = 90°

Find : m ($\angle$ E)

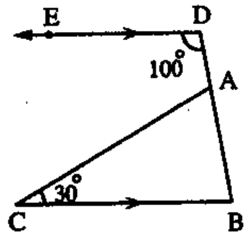


36. **In the opposite figure :**

$\overrightarrow{DE}$ // $\overline{BC}$, m ($\angle$ D) = 100°

, m ($\angle$ C) = 30° and A $\in \overline{DB}$

Find : m ($\angle$ BAC)



37. **Using the square lattice :**

Draw Δ ABC , where A (– 2 , 3) , B (2 , 3) , C (2 , 6) , then

Find its image by translation $(x , y) \longrightarrow (x + 2 , y - 1)$

38. Draw Δ OBC on a square lattice where O (0 , 0) , B (3 , 0) , C (0 , 4) , then draw its image by rotation about the origin point with an angle of measure 180°

39. Draw the image of triangle ABC where A (1 , 1) , B (3 , 4) , C (5 , 2) by reflection in x-axis.

40. Using the square lattice , draw $\overline{AB}$ where A (4 , 3) , B (– 1 , 1) then find the image of $\overline{AB}$ by translation $(x , y) \longrightarrow (x + 2 , y - 1)$

SECOND: GEOMETRY

Choose the correct answer :

1.	The image of the point (2 , – 1) by reflection in y-axis is .... (a) (2 , 1) (b) (– 2 , – 1) (c) (– 2 , 1) (d) (2 , – 1)
2.	ABCD is a parallelogram , m (∠ A) = 70° , then m (∠ C) = ... ° (a) 110 (b) 35 (c) 70 (d) 140
3.	The diagonals are equal in length and perpendicular in .... (a) square. (b) rhombus. (c) rectangle. (d) parallelogram.
4.	The image of the point (5 , – 3) by translation 3 units in negative direction of X-axis is .... (a) (– 3 , 5) (b) (– 2 , – 3) (c) (2 , – 3) (d) (5 , 0)
5.	In Δ ABC , if D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , BC = 8 cm. , then DE = ... cm. (a) 16 (b) 8 (c) 4 (d) 2
6.	✱ The triangle contains two .... angles at least (a) acute (b) obtuse (c) right (d) reflex
7.	The image of the point (– 3 , 5) by rotation about the origin point and with an angle of measure 90° is .... (a) (5 , 3) (b) (– 5 , 3) (c) (3 , 5) (d) (– 5 , – 3)
8.	The image of the point (2 , – 1) by reflection in X-axis is .... (a) (2 , 1) (b) (2 , 2) (c) (– 2 , – 1) (d) (– 1 , 2)
9.	In Δ ABC if : m (∠ A) > m (∠ B) + m (∠ C) , then the angle A is .... (a) acute. (b) right. (c) obtuse. (d) straight.

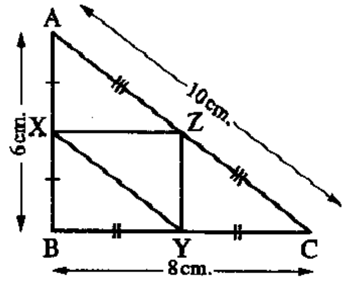
10.	The image of the square by rotation about origin point with an angle of measure 90° is .... (a) rectangle. (b) square. (c) rhombus. (d) trapezium.
11.	In Δ ABC if : X , Y are the midpoints of $\overline{AC}$ and $\overline{BC}$ respectively , then $\overline{XY}$ // .... (a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{AC}$ (d) $\overline{CY}$
12.	The measure of the interior angle of a regular polygon of 18 sides equals .... (a) 130° (b) 140° (c) 150° (d) 160°
13.	The image of the point (2 , – 5) by reflection in X-axis is .... (a) (– 2 , 5) (b) (2 , 5) (c) (– 2 , – 5) (d) (5 , 2)

Complete each of the following :

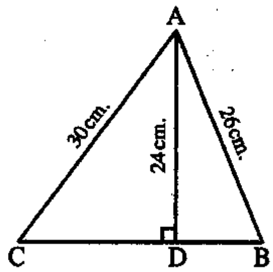
1.	The ray drawn from the midpoint of a side of the triangle parallel to another side ....
2.	The number of axis of symmetry of isosceles triangle = ...
3.	The image of the point .... by rotation about the origin point with an angle of measure 90° is (– 1 , 4)
4.	The length of diagonal of a rectangle whose dimensions are 6 cm. and 8 cm. = ... cm.
5.	The translation is determined by .... and ....
6.	The line segment joining the midpoints of two sides of a triangle is .... the third side.
7.	The image of the point (3 , – 2) by translation on (X – 1 , y + 6) is ....
8.	The image of the point (3 , 2) by rotation with an angle of measure 180° about the origin point is ....
9.	The image of the point (4 , 1) by reflection in the origin point is ....
10.	In Δ XYZ , m (∠ Y) = 90° , then $(XZ)^2$ = ...
11.	If the point (1 , 4) is the image of the point (– 1 , 3) by a translation (X , y) , then the image of the point (3 , – 2) by the same translation is ....

Essay problems:

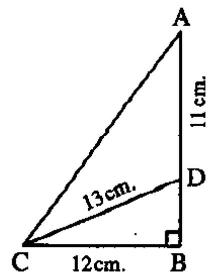
1. **In the opposite figure :**
   X , Y , Z are midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$, AB = 6 cm. , BC = 8 cm. , AC = 10 cm.
   Find : the perimeter of Δ XYZ



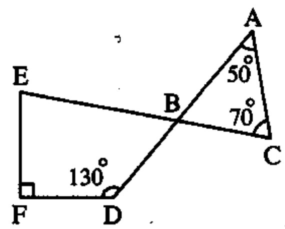
2. **In the opposite figure :**
   $\overline{AD} \perp \overline{BC}$, if AD = 24 cm. , AB = 26 cm. , AC = 30 cm.
   Find :
   ① The length of $\overline{BC}$
   ② The area of Δ ABC



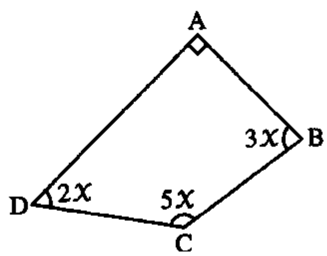
3. **In the opposite figure :**
   Δ ABC in which m (∠ B) = 90°
   D ∈ $\overline{AB}$ such that AD = 11 cm.
   , if BC = 12 cm , DC = 13 cm.
   Find the length of each of : $\overline{AB}$, $\overline{AC}$



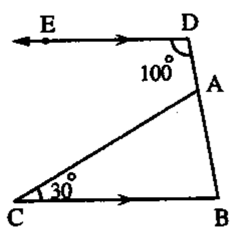
4. **✱ In the opposite figure :**
   $\overline{CE} \cap \overline{AD} = \{B\}$, m (∠ A) = 50° , m (∠ C) = 70° , m (∠ D) = 130° and m (∠ F) = 90°
   Find : m (∠ E)



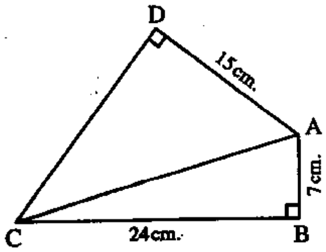
5. **In the opposite figure :**
   ABCD is a quadrilateral in which : m (∠ A) = 90°
   Find : the value of x



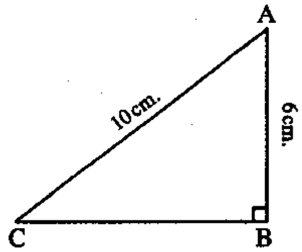
6. **In the opposite figure :**
   $\overrightarrow{DE}$ // $\overline{BC}$, m (∠ D) = 100° , m (∠ C) = 30° and A ∈ $\overline{DB}$
   Find : m (∠ BAC)



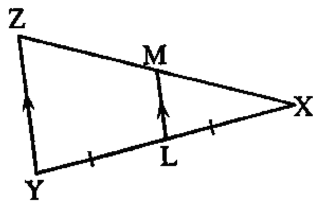
7. **In the opposite figure :**
m (∠ B) = m (∠ D) = 90° , AD = 15 cm.
, AB = 7 cm. , BC = 24 cm.
Find : the length of $\overline{AC}$, $\overline{DC}$



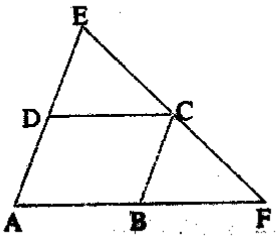
8. **In the opposite figure :**
ABC is a triangle in which
m (∠ B) = 90° , AB = 6 cm. , AC = 10 cm.
Find with proof : the length of $\overline{BC}$



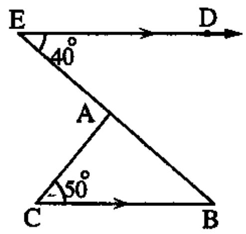
9. **In the opposite figure :**
L is a midpoint of $\overline{XY}$
, $\overline{LM}$ // $\overline{YZ}$, XZ = 10 cm.
Find : the length of $\overline{XM}$



10. **In the opposite figure :**
ABCD is a parallelogram
E ∈ $\overrightarrow{AD}$ where AD = DE , $\overrightarrow{EC}$ ∩ $\overrightarrow{AB}$ = {F}
Prove that : EC = CF



11. **In the opposite figure :**
$\overrightarrow{ED}$ // $\overrightarrow{CB}$, m (∠ C) = 50°
, m (∠ E) = 40°
Prove that : $\overline{AC} \perp \overline{BE}$



12. Draw Δ ABC where A (1 , 5) , B (3 , 1) and C (5 , 3) , **then draw its image :**
① By reflection in y-axis.
② By rotation about origin point with an angle of measure 180°

13. In a cartesian plane draw the image of Δ ABC where A (– 2 , 3) , B (2 , 3) , C (2 , 6)
by translation $(x , y) \longrightarrow (x + 2 , y - 1)$

14. Using the square lattice , draw the triangle ABC where A (3 , – 1) , B (5 , 2)
and C (– 2 , 4) , then draw its image by rotation R (O , 180°)

Geometry

choose:

① b	② a	③ d	④ b
⑤ b	⑥ b	⑦ c	⑧ b
⑨ c	⑩ c	⑪ c	⑫ b
⑬ b	⑭ a	⑮ b	⑯ a
⑰ b	⑱ b	⑲ c	⑳ c
㉑ b	㉒ c	㉓ b	㉔ d
㉕ d	㉖ c	㉗ d	㉘ a
㉙ b	㉚ a	㉛ c	㉜ c
㉝ b	㉞ d	㉟ c	㊱ a
㊲ a	㊳ b	㊴ d	㊵ c

Complete:

① (2, −1) ② (−2, 1)
③ The diagonals are perpendicular
④ 120° ⑤ (8, 2) ⑥ 360°
⑦ (5, 5) ⑧ 120°
⑨ bisects the third side
⑩ half ⑪ right ⑫ 6
⑬ (3, −2) ⑭ 70° ⑮ (4, 3)
⑯ 3, 3 ⑰ $\pm 180^\circ$ ⑱ 125°
⑲ 11 ⑳ measure ㉑ Parallel
㉒ the sum of areas of squares
㉓ 360° ㉔ (−3, 8) ㉕ $(BD)^2$
㉖ rhombus ㉗ 6
㉘ (2, −1) ㉙ (4, 2) ㉚ 108°
㉛ square ㉜ 120°
㉝ Parallel to the third side
㉞ (−3, −7) ㉟ 12 ㊱ $\overline{AC} \perp \overline{BD}$
㊲ (−2, −1) ㊳ 360°
㊴ 15 ㊵ Parallel

Essay:

① $\because$ ($\angle ACD$) is an exterior
$\therefore m(\angle ACD) = m(\angle A) + m(\angle B)$
$= 25 + 25 = 50^\circ$

② $\because$ E, D are midpoint of $\overline{AB}, \overline{AC}$
$\therefore ED = \frac{1}{2} BC$ $\therefore ED = 5\text{cm}$
$\therefore P. \triangle AED = 5 + 6 + 4 = 15$ cm

③ $\because \overline{ED} \parallel \overline{ZY}$
$\therefore m(\angle D) = m(\angle Z) = 50^\circ$ Alt.
$\therefore m(\angle Y) = 180 - (105 + 50) = 25^\circ$
$\because$ ($\angle YXD$) is an exterior
$\therefore m(\angle YXD) = m(\angle Z) + m(\angle Y)$
$= 50 + 25 = 75^\circ$

④ $\because \overline{AZ} \parallel \overline{YD} \parallel \overline{XE} \parallel \overline{CB}$,
$AY = YX = XC$
$\therefore AD = DE = EB = \frac{18}{3} = 6\text{cm}$

⑤ In $\triangle ADB$, $\because m(\angle D) = 90^\circ$
$\therefore (BB)^2 = (AB)^2 - (AD)^2 = 100$
$\therefore DB = 10$ cm
In $\triangle ADC$, $\because m(\angle D) = 90^\circ$
$\therefore (CD)^2 = (AC)^2 - (AD)^2 = 324$
$\therefore CD = \sqrt{324} = 18$ cm
$\therefore BC = 10 + 18 = \boxed{28 \text{ cm}}$
Area $\triangle ABC = \frac{1}{2} \times 28 \times 24 = 336$ cm^2

⑥ $\because$ ABCD is a square
$\therefore \overline{AD} \parallel \overline{BC}$, $\because E \in \overrightarrow{BC}$
$\therefore \overline{EC} \parallel \overline{AD}$, $\because \overline{AC} \parallel \overline{DE}$
$\therefore$ ACED is a parallelogram.

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⑦ In ΔXYZ, $\because m(\angle Y) = 90°$

$\therefore (XZ)^2 = (XY)^2 + (YZ)^2 = 625$

$\therefore XZ = \sqrt{625} = \boxed{25\text{ cm}}$

In ΔXLZ, $\because m(\angle L) = 90°$

$\therefore (LZ)^2 = (XZ)^2 - (LX)^2 = 400$

$\therefore LZ = \sqrt{400} = \boxed{20\text{ cm}}$

⑧ In ΔABC

$m(\angle ACB) = 180 - (30 + 90) = 60°$

$\therefore m(\angle ACB) = m(\angle DCO) = 60°$ (V.O.A.)

$\therefore m(\angle E) = 360 - (60 + 120 + 90)$
$= 90°$

⑨ $\because \overrightarrow{ED} \parallel \overline{CD}$

$\therefore m(\angle E) = m(\angle ABC) = 50°$ Alt.

$m(\angle BAC) = 180 - (30 + 50)$
$= 100°$

$\because (\angle ABD)$ is an exterior

$\therefore m(\angle ABD) = 100 + 30 = 130°$

⑩ $\because \overline{DA} \parallel \overline{YX} \parallel \overline{CB}$ and $AX = XB$

$\therefore DY = YC$

In ΔCDE

$\because Y$ is midpoint, $\overline{YZ} \parallel \overline{DE}$

$\therefore YZ$ is the midpoint of $\overline{EC}$

$\therefore CZ = ZE$

⑪ Translation $= N - M = (5, 2)$

$A(2,1) \xrightarrow{(5,2)} A'(7,3)$

$B(2,4) \xrightarrow{(5,2)} B'(7,6)$

and draw the points on the Coordinates plane.

⑫ $\because F, D$ are midpoints of $\overline{AC}, \overline{AB}$

$\therefore FD = \frac{1}{2} BC = 8\text{ cm}$

$\because D, E$ are midpoints of $\overline{AB}, \overline{BC}$

$\therefore DE = \frac{1}{2} AC = 7\text{ cm}$

$\because F, E$ are midpoints of $\overline{AC}, \overline{BC}$

$\therefore FE = \frac{1}{2} AB = 5\text{ cm}$

$\therefore$ P. of $\Delta DEF = 8 + 7 + 5 = 20\text{ cm}$

⑬ as no. (7)

⑭ as no. (5)

⑮ as no. (2)

⑯ as no. (12)

⑰ In ΔFED

$\because \Delta FED$ is an equilateral

$\therefore$ the measure of each angle $= 60°$

$\therefore m(\angle FDE) = 60°$

$\therefore m(\angle ADC) = m(\angle FDE) = 60°$ (V.O.A.)

$\therefore m(\angle B) = 360 - (120 + 105 + 60)$
$= 75°$

⑱ In ΔDBC

$\because O, H$ are midpoints of $\overline{DC}, \overline{DB}$

$\therefore \overline{OH} \parallel \overline{BC}$, $OH = \frac{1}{2} BC$

$\because \overline{AD} \parallel \overline{BC}$, $AD = \frac{1}{2} BC$

$\therefore \overline{AD} \parallel \overline{OH}$, $AD = OH$

$\therefore AHOD$ is a parallelogram.

(19) $\because m(\angle x) = 90^\circ$

$\therefore (Yx)^2 = (ZY)^2 - (XZ)^2 = 144\ cm^2$

$\therefore Yx = \sqrt{144} = 12\ cm$

(20) $\because (\angle AMB)$ exterior of ΔMBC

$\therefore m(\angle MCB) = 70 - 40 = 30^\circ$

$\because m(\angle DAC) = m(\angle ACB)$

But they are alternate

$\therefore \overline{AD} \parallel \overline{BC}$, $\because \overline{AB} \parallel \overline{DC}$

$\therefore ABCD$ is a parallelogram.

(21) In ΔABD, $\because m(\angle A) = 90^\circ$

$\therefore (BD)^2 = (AD)^2 + (AB)^2 = 625\ cm^2$

$\therefore BD = \sqrt{625} = 25\ cm$

In ΔBCD, $\because m(\angle C) = 90^\circ$

$\therefore (BC)^2 = (BD)^2 - (CD)^2 = 49\ cm^2$

$\therefore BC = \sqrt{49} = 7\ cm$

$\therefore P = 7 + 24 + 25 = 56\ cm$

(22) As no. (12)

(23) $\because \overline{XY} \perp \overleftrightarrow{YZ}$, $\overline{LM} \perp \overleftrightarrow{YZ}$

$\therefore \overline{XY} \parallel \overline{LM}$, $\because \overline{XL} \parallel \overrightarrow{YM}$

$\therefore XLMY$ is a rectangle

$\therefore XL = YM = 9\ cm$

$\therefore MZ = 17 - 9 = 8\ cm$

In ΔLMZ, $\because m(\angle M) = 90^\circ$

$\therefore (LM)^2 = (LZ)^2 - (MZ)^2 = 225\ cm^2$

$\therefore LM = \sqrt{225} = 15\ cm$

$\therefore$ $\boxed{XY = 15\ cm}$

The image is $\overline{YM}$

(24) $\because ABCD$ is a parallelogram

$\therefore M$ is the midpoint of $\overline{AC}$

In ΔABC

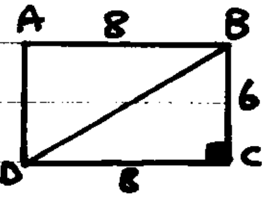
$\because M$ is the mid point of $\overline{AC}$

and $\overline{ME} \parallel \overline{AB}$

$\therefore E$ is the midpoint of $\overline{BC}$

$\therefore BE = EC$.

(25)



$L = A \div w = 48 \div 6$

$\therefore AB = 8\ cm$

In ΔBCD, $\because m(\angle C) = 90^\circ$

$\therefore (BD)^2 = (BC)^2 + (DC)^2 = 100\ cm^2$

$\therefore BD = \sqrt{100} = 10\ cm$

(26) $\because \overrightarrow{ED} \parallel \overline{BC}$

$\therefore m(\angle E) = m(\angle B) = 40^\circ$ Alt.

$\therefore m(\angle BAC) = 180 - (40 + 50)$

$= 90^\circ$

$\therefore \overline{AC} \perp \overline{EB}$

(27) $\because \overline{BC} \parallel \overline{DE}$

$\therefore m(\angle B) = m(\angle BAE) = 120^\circ$ Alt.

$\because m(\angle C) + m(\angle B) = 180^\circ$

But they are interior

$\therefore \overline{DC} \parallel \overline{AB}$, $\because \overline{BC} \parallel \overline{AD}$

$\therefore ABCD$ is a parallelogram

(28) As no. (12)

(29) $\because m(\angle B) = 90^\circ$

$\therefore (AB)^2 = (AC)^2 - (BC)^2 = 36\ cm^2$

$\therefore AB = \sqrt{36} = 6\ cm$

(30) In $\triangle ABE$

$m(\angle B) = 180 - (45 + 70) = 65^\circ$

In quad. ABCD

$m(\angle DAB) = 360 - (65 + 115 + 65)$

$= 115^\circ$

$\because m(\angle D) = m(\angle B)$ and

$m(\angle A) = m(\angle C)$

$\therefore$ ABCD is a parallelogram

(31) $\because$ L is the midpoint of $\overline{XY}$

and $\overleftrightarrow{LM} \parallel \overline{YZ}$

$\therefore$ M is the midpoint of $\overline{ZX}$

$\therefore XM = 5$ cm

(32) $\because$ ABCD is a rhombus

$\therefore m(\angle ABD) = m(\angle ADB) = 62^\circ$

$\therefore m(\angle A) = 180 - (62 + 62) = 56^\circ$

(33) $2a + a + a + 2a + 2a = 540^\circ$

$\therefore 8a = 540^\circ \quad \therefore a = 67.5^\circ$

(34) In $\triangle DBC$, $\because m(\angle B) = 90^\circ$

$\therefore (BD)^2 = (CD)^2 - (BC)^2 = 25 \text{ cm}^2$

$\therefore BD = \sqrt{25} = 5$ cm

$\therefore AB = 5 + 11 = \boxed{16 \text{ cm}}$

In $\triangle ABC$, $\because m(\angle B) = 90^\circ$

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 400 \text{ cm}^2$

$\therefore AC = \sqrt{400} = 20$ cm

(35) $m(\angle ABC) = 180 - (50 + 70) = 60^\circ$

$\therefore m(\angle ABC) = m(\angle EBD) = 60^\circ$ (V.O.A.)

$\therefore m(\angle E) = 360 - (60 + 130 + 90)$

$= 80^\circ$

(36) $\because \overline{DE} \parallel \overline{BC}$

$\therefore m(\angle B) = 180 - 100 = 80^\circ$ (interior)

$\therefore m(\angle BAC) = 180 - (80 + 30) = 70^\circ$

(37) $A(-2,3) \xrightarrow{(2,-1)} A'(0,2)$

$B(2,3) \longrightarrow B'(4,2)$

$C(2,6) \longrightarrow C'(4,5)$

(38) $O(0,0) \xrightarrow{R(O,180)} O'(0,0)$

$B(3,0) \longrightarrow B'(-3,0)$

$C(0,4) \longrightarrow C'(0,-4)$

(39) $A(1,1) \xrightarrow[\text{Refl.}]{x\text{-axis}} A'(1,-1)$

$B(3,4) \longrightarrow B'(3,-4)$

$C(5,2) \longrightarrow C'(5,-2)$

(40) $A(4,3) \xrightarrow{(2,-1)} A'(2,2)$

$B(-1,1) \longrightarrow B'(1,0)$

Algebra

Choose:

① c ② b ③ c ④ a

⑤ d ⑥ c ⑦ b ⑧ c

⑨ b ⑩ c ⑪ b ⑫ c

⑬ b ⑭ d ⑮ a

Complete:

① ϕ ② 7cm ③ 20 ④ 2

⑤ $\{x : x \in \mathbb{Q}, x \geq \frac{5}{3}\}$ ⑥ 2

⑦ 0 ⑧ 1 ⑨ 5 ⑩ $\frac{31}{32}, \frac{63}{64}$

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Essay problems:

① $x = \frac{8-3}{3}$, $x = \frac{5}{3}$, S.S. $= \{\frac{5}{3}\}$

② $x < \frac{1-3}{-2}$, $x \leq 1$, S.S. $= \{1, 0\}$

③ $1 \times \frac{4}{5} \times \frac{5}{2} = 2$

④ ① $\frac{6}{15} = \frac{2}{5}$ ② $\frac{5}{15} = \frac{1}{3}$ ③ $\frac{3}{15} = \frac{1}{5}$

⑤ $x^2 = 81$, $x = \pm 9$

⑥ $x = \frac{-5-3}{-4}$, $x = 2$, S.S. $= \{2\}$

⑦ ① $\frac{5}{10} = \frac{1}{2}$ ② $\frac{1}{10}$

⑧ $x < \frac{16-6}{2}$, $x < 5$
S.S. $= \{x : x \in Q , x < 5\}$

⑨ $5x - 2x \geq 11 + 4$, $3x \geq 15$
$x \geq 5$, S.S. $= \{x : x \in Q , x \geq 5\}$

⑩ $(2^{5-3} \times 3^{2-4})^{-1} = (2^{2} \times 3^{-2})^{-1} = (\frac{4}{9})^{-1} = \frac{9}{4}$

⑪ $\frac{1}{9} + \frac{8}{9} + 1 = 1 + 1 = 2$

⑫ $(9^{3+1-5})^{-3} = (9^{-1})^{-3} = 9^{3} = 729$

⑬ Absent $= 0.15 \times 600 = 90$ students.
present $= 600 - 90 = 510$ students.

⑭ $\frac{\not{4}(2)^{5} \times (2)^{4}}{\not{4}(2)^{3} \times 2^{2}} = 2^{5+4-3-2} = 2^{4} = 16$

⑮ $\frac{1}{9} + \frac{8}{9} + 1 = 1 + 1 = 2$

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Geometry

choose:

① b ② c ③ a ④ c
⑤ c ⑥ a ⑦ d ⑧ a
⑨ c ⑩ b ⑪ a ⑫ d
⑬ b

complete:

① bisects the third side
② 1 ③ (4,1) ④ 10
⑤ magnitude and direction
⑥ Parallel ⑦ (2,4) ⑧ (-3,-2)
⑨ (-4,-1) ⑩ $(XY)^2 + (YZ)^2$
⑪ (5,-1)

Essay problems:

① As no. (12) P.(4)

② As no. (5) P.(3)

③ As no. (34) P. (6)

④ As no. (35) P.(6)

⑤ $3x + 5x + 2x + 90 = 360$
$\therefore 10x = 270$, $\boxed{x = 27^\circ}$

⑥ As no. (36) P.(6)

⑦ As no.(7) P. (4)

⑧ $\because m(\angle B) = 90^\circ$
$\therefore (BC)^2 = (AC)^2 - (AB)^2 = 64 \text{ cm}^2$
$\therefore BC = \sqrt{64} = 8 \text{ cm}$

⑨ As no.(31) P.(6)

⑩ ∵ ABCD is a parallelogram

∴ $\overleftrightarrow{DC} \parallel \overleftrightarrow{AB}$

∵ AD = DE

∴ D is the midpoint of $\overline{EA}$

∴ C is the midpoint of $\overline{EF}$

∴ EC = FC

⑪ As no.(26) P.(5)

⑫ $A(1,5) \xrightarrow[\text{y-axis}]{\text{Ref.}} A'(-1,5)$

$B(3,1) \longrightarrow B'(-3,1)$

$C(5,3) \longrightarrow C'(-5,3)$

$A(1,5) \xrightarrow{R(O,180^\circ)} A''(-1,-5)$

$B(3,1) \longrightarrow B''(-3,-1)$

$C(5,3) \longrightarrow C''(-5,-3)$

⑬ $A(-2,3) \xrightarrow{(2,-1)} A'(0,2)$

$B(2,3) \longrightarrow B'(4,2)$

$C(2,6) \longrightarrow C'(4,5)$

⑭ $A(3,-1) \xrightarrow{R(O,180^\circ)} A'(-3,1)$

$B(5,2) \longrightarrow B'(-5,-2)$

$C(-2,4) \longrightarrow C'(2,-4)$

Best wishes

إعداد/ محمود أحمد محب

GEOMETRY

Relations between the angles

1 TYPES OF ANGLES :

Acute : Its measure less than 90° and more than 0°

Zero : Its measure = 0°

Obtuse : Its measure less than 180° and more than 90°

Right : Its measure = 90°

Reflex : Its measure less than 180° and more than 90°

Straight : Its measure = 180°

2 TYPES OF ANGLES :

Two adjacent angles : They have a common vertex and a common side and the other two sides are on opposite sides of this common side

Complementary Angles : The sum of their measures is 90°

Supplementary Angles : The sum of their measures is 180°

Two adjacent Supplementary Angles : The two adjacent angles formed by a ray with a starting point on this straight line , are supplementary.

Vertically opposite angles (V.O.A) : If two straight lines intersect , then the measures of each two vertically opposite angles are equal in measure.

Accumulative angles at a point : the sum of the measures of the accumulative angles at a point = 360°.

3 REMARKS :

1. If the two outer sides of two adjacent angles are perpendicular , then these two adjacent angles Are complementary
2. If the two outer sides of two adjacent angles are On the same straight lines , then these two adjacent angles Are supplementary
3. If the two adjacent angles are supplementary , then their outer sides are collinear.
4. The acute angle supplements an obtuse angle and complements an acute angle
5. The Zero angle supplements a straight angle and complements a right angle
6. The right angle supplements a right angle and complements a zero angle
7. The two bisectors of two adjacent complementary angles are perpendicular
8. the angle of measure x° complements an angle of measure ($90^\circ - x^\circ$)
9. the angle of measure x° supplements an angle of measure ($180^\circ - x^\circ$)
10. The measure of the reflex angle of the angle whose measure x° equals ($360^\circ - x^\circ$)

School book

Important Exercises

1 Which of the following is a Right angle ?

ⓐ 60° ⓑ 30° ⓒ $89^\circ 60'$ ⓓ 180°

2 the angle of measure 54° complements an angle of measure ....

ⓐ 36° ⓑ 126° ⓒ 304° ⓓ 90°

3 the angle of measure 75° supplements an angle of measure ....

ⓐ 15° ⓑ 105° ⓒ 285° ⓓ 180°

4 If : m (∠ A) = 70° , then : m (reflex ∠ A) = ...

ⓐ 110° ⓑ 20° ⓒ 180° ⓓ 290°

5 The acute angle supplements .... angle

ⓐ an acute ⓑ a right ⓒ an obtuse ⓓ a zero

6 If the ratio between two complementary angles is 2 : 7 , then the measure of the smallest = ...

ⓐ 90° ⓑ 20° ⓒ 70° ⓓ 180°

7 If two straight lines intersects , then each of two vertically opposite angles are ....

ⓐ complementary ⓑ supplementary ⓒ alternate ⓓ congruent

8 The sum of the measures of the accumulative angles at the point equals ....

ⓐ 90° ⓑ 180° ⓒ 270° ⓓ 360°

9 The measure of the straight angle = ...

ⓐ 90° ⓑ 180° ⓒ 60° ⓓ 360°

10 The angle whose measure is 89° is .... angle.

ⓐ an acute ⓑ an obtuse ⓒ a zero ⓓ a reflex

11 If : m (∠ A) + m (∠ B) = 180° , then : ∠ A and ∠ B Are ....

ⓐ complementary ⓑ supplementary ⓒ alternate ⓓ congruent

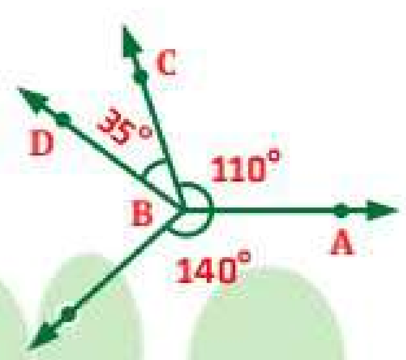
12 The number of axis of the rectangle = ...

ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 4

13 In the opposite Figure :

m (∠ ABC) = 110° , m (∠CBD) = 35° and m (∠ ABE) = 140°

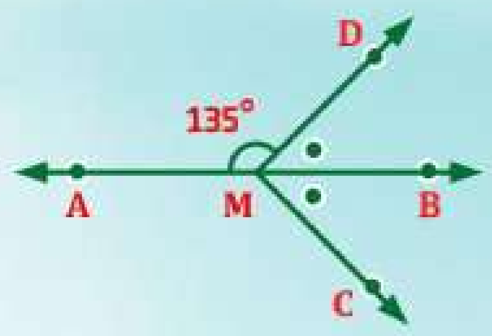
, Find m (∠ EBD)



14 In the opposite Figure :

$\overrightarrow{MB}$ bisects (∠ DMC) , m (∠ AMD) = 135 °

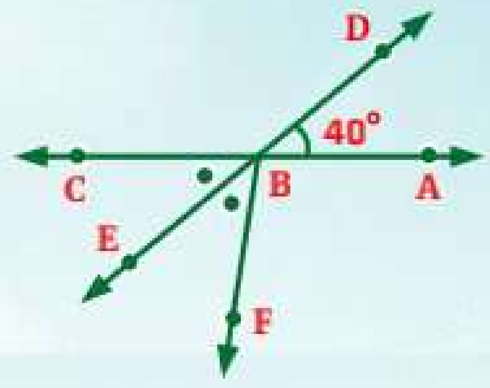
Find : m (∠ CMD) , m (∠ AMC)



15 In the opposite Figure :

$\overleftrightarrow{AC} \cap \overleftrightarrow{DE}$ = { B } , m (∠ ABD) = 40 ° and $\overrightarrow{BE}$ bisects (∠ CBF)

Find m (∠ ABF)



Congruent

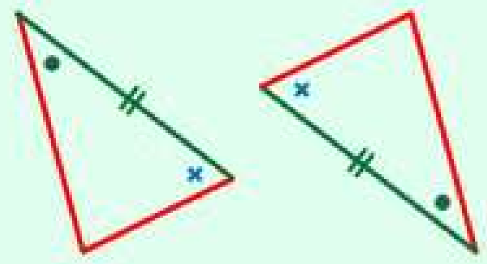
1 **THE CONGRUENCE OF TWO POLYGONS :**

Two polygons are congruent if :

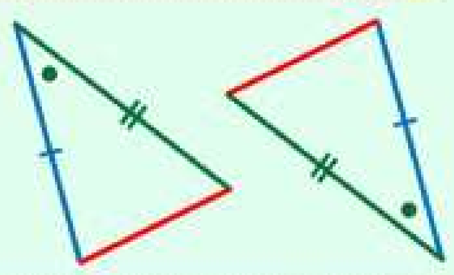
1. each two corresponding sides are (congruent) equal in length
2. each two corresponding angles are (congruent) equal in measure

2 **THE CONGRUENCE OF TWO TRIANGLE :**

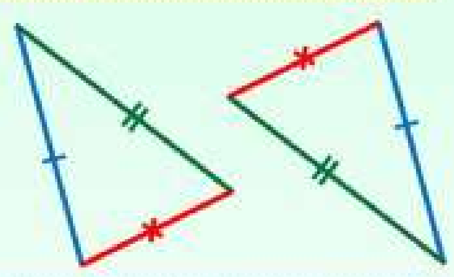
First case : The two triangles are congruent if two angles and one side in one of the triangles are congruent to their corresponding elements in the other.



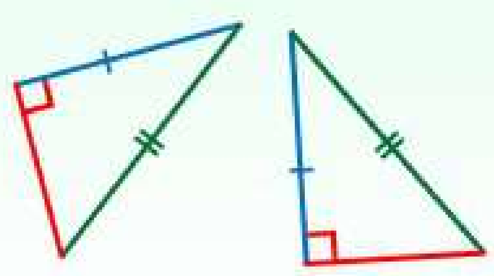
Second case : The two triangles are congruent if two sides and the included angle in one of the triangles are congruent to their corresponding elements in the other.



Third case : The two triangles are congruent if each side in one of the triangles are congruent to their corresponding elements in the other.



Fourth case : The two right angled triangles are congruent if the hypotenuse and one Side of one triangle are congruent to their corresponding elements in the other.



School book

Important Exercises

1 If : $\overline{AB} \equiv \overline{XY}$, then : AB = ...

ⓐ XY ⓑ $\overline{XY}$ ⓒ $\overrightarrow{XY}$ ⓓ $\overleftrightarrow{XY}$

2 If : $\overline{AB} \equiv \overline{XY}$, then : AB – XY = ...

ⓐ 2 AB ⓑ 2 XY ⓒ zero ⓓ 2

3 If : ∠ A supplements ∠ B and ∠ A ≡ ∠ B ,then : m (∠ A) = ...

ⓐ 45 ° ⓑ 90 ° ⓒ 50 ° ⓓ 60 °

4 If : Δ ABC ≡ Δ XYZ , then : AB = ...

ⓐ BC ⓑ XZ ⓒ XY ⓓ YZ

5 If : $\Delta ABC \equiv \Delta XYZ$, then : AB = ...

ⓐ BC ⓑ XZ ⓒ XY ⓓ YZ

6 If : $\Delta ABC \equiv \Delta XYZ$, $m(\angle A) = 50°$, $m(\angle Y) = 60°$, then : $m(\angle Z) =$ ....

ⓐ 50° ⓑ 60° ⓒ 70° ⓓ 110°

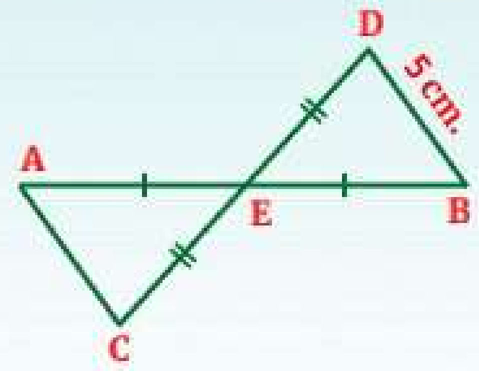
7 If : $\Delta ABC \equiv \Delta XYZ$, $m(\angle A) + m(\angle B) = 130°$, then : $m(\angle Z) =$ ....

ⓐ 65° ⓑ 70° ⓒ 130° ⓓ 50°

8 **In the opposite Figure :**

Is $\Delta ACE \equiv \Delta BDE$, Why ?

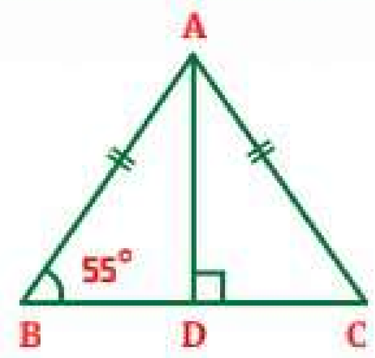
, Find the length of $\overline{AC}$



8 **In the opposite Figure :**

AB = AC , $m(\angle B) = 55°$

Prove That $\Delta ABD \equiv \Delta ACD$, Then Find $m(\angle CAD)$



Parallelism

1 PARALLELISM

If two parallel straight lines are intersected by a third , then :

1. Each two alternate angles are equal in measure
2. Each two corresponding angles are equal in measure
3. Each two interior angles in the same side of the transversal are supplementary

2 FACTS :

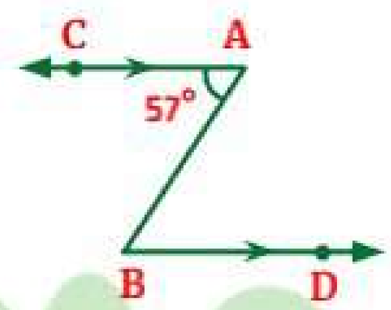
1. The perpendicular to one of Two parallel straight lines is Perpendicular to the other in the plane.
2. If two straight lines are parallel to a third straight line , then they are Parallel
3. If two straight lines are Perpendicular to a third one in the same plane , then they are Parallel
4. If parallel straight lines divide a straight line into segments of equal lengths , then they divide any other straight line into segments of equal lengths .

School book

Important Exercises

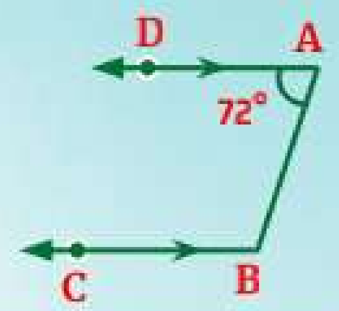
1 **In the opposite Figure :**

$m(\angle ABC) =$ ....



ⓐ 33° ⓑ 57° ⓒ 43° ⓓ 123°

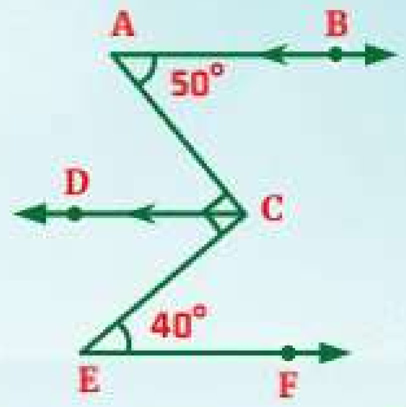
2 In the opposite Figure :



m (∠ ABC) = ...

ⓐ 72 ° ⓑ 108 ° ⓒ 18 ° ⓓ 180 °

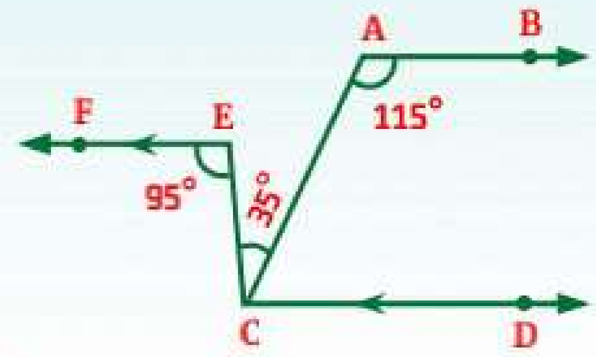
3 In the opposite Figure :



$\overrightarrow{AB}$ // $\overrightarrow{CD}$, m (∠ A) = 50 ° , m (∠ E) = 40 ° and ∠ ACE is right

Prove that : $\overrightarrow{AB}$ // $\overrightarrow{EF}$

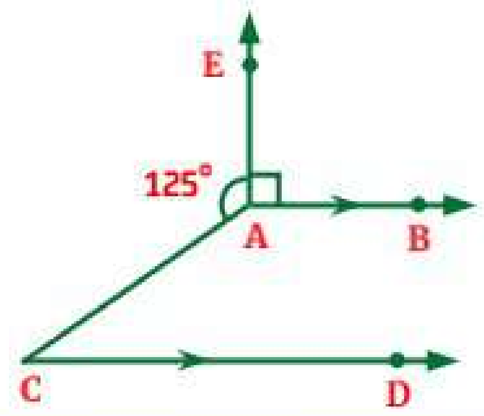
4 In the opposite Figure :



$\overrightarrow{EF}$ // $\overrightarrow{CD}$, m (∠ CEF) = 95 ° , m (∠ ACE) = 30 ° and , m (∠ BAC) = 115 °

Prove that : $\overrightarrow{AB}$ // $\overrightarrow{EF}$

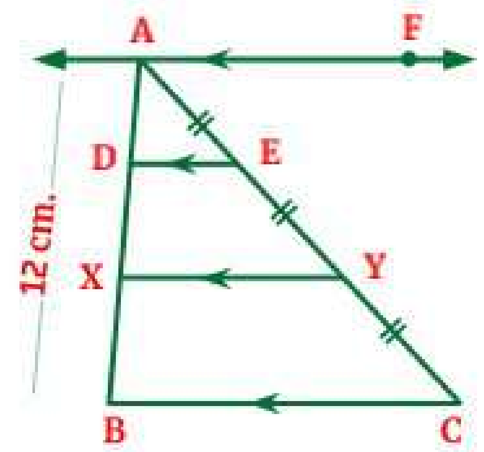
5 In the opposite Figure :



$\overrightarrow{AB}$ // $\overrightarrow{CD}$, m (∠ EAC) = 125 ° , m (∠ EAB) = 90 °

Find m (∠ C)

6 In the opposite Figure :



$\overline{AF}$ // $\overline{DE}$ // $\overline{XY}$ // $\overline{BC}$, AE = EY = YC and AB = 12 cm.

Find the length of $\overline{AX}$

Polygons

1 **POLYGONS :**

1. The sum of measures of the interior angles of a polygon of n sides = (n – 2) × 180°
2. The sum of measures of the exteriors angles of a polygon of n sides = 360°
3. The measure of each interior angle of the regular polygon of n sides = $\frac{(n-2)\times 180^\circ}{n}$
4. The measure of each exterior angle of the regular polygon of n sides = $\frac{360^\circ}{n}$
5. The number of diagonals of the polygon of n sides = $\frac{n(n-3)}{2}$
6. The number of sides of the regular polygon in which the measure of one of the interior angle is x ° = $\frac{360^\circ}{180^\circ - x}$
7. The perimeter of the regular polygon whose side length = L and of n sides = n L

School book

Important Exercises

1 The sum of measure of the interior angles of the quadrilateral = ...

ⓐ 180° ⓑ 360° ⓒ 540° ⓓ 720°

2 The sum of measure of the interior angles of the octagon = ...

ⓐ 540° ⓑ 720° ⓒ 900° ⓓ 1080°

3 The measure of the interior angle of the regular hexagon = ...

ⓐ 90° ⓑ 120° ⓒ 60° ⓓ 135°

4 The measure of the interior angle of the regular pentagon = ...

ⓐ 60° ⓑ 90° ⓒ 108° ⓓ 120°

5 The sum of measure of the exterior angles of the hexagon = ...

ⓐ 180° ⓑ 360° ⓒ 540° ⓓ 720°

6 The measure of the exterior angle of the regular octagon = ...

ⓐ 45° ⓑ 60° ⓒ 120° ⓓ 135°

7 If the measure of an interior angle of a regular polygon is 140°, then the number of its sides = ...

ⓐ 8 ⓑ 9 ⓒ 10 ⓓ 11

8 The number of diagonals of the triangle = ...

ⓐ 0 ⓑ 1 ⓒ 2 ⓓ 3

9 The number of diagonals of the hexagon = ...

ⓐ 3 ⓑ 6 ⓒ 9 ⓓ 10

10 If the perimeter of a regular hexagon is 60 cm. , then its side length = ...

ⓐ 6 ⓑ 10 ⓒ 30 ⓓ 15

11 If the interior angle of a regular hexagon is $(3x + 25)^\circ$, then : $6x$ = ...

ⓐ 120 ⓑ 95 ⓒ 190 ⓓ 180

12 which of the following can be a measure of an interior angle of a regular polygon ?

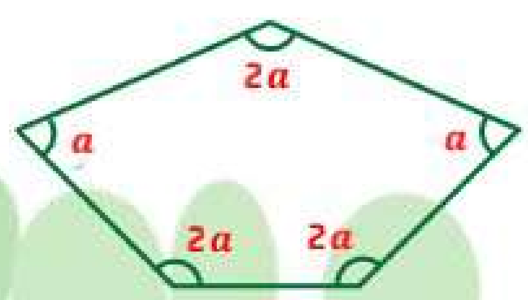
ⓐ 50 ⓑ 60 ⓒ 100 ⓓ 150

13 The measure of the exterior angle of the regular hexagon = ...

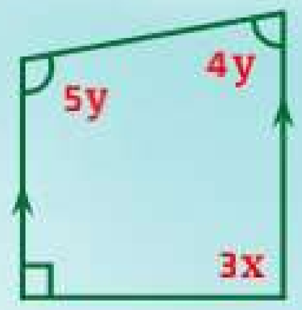
ⓐ 60° ⓑ 108° ⓒ 120° ⓓ 135°

14 In the opposite Figure :

Find the value of a

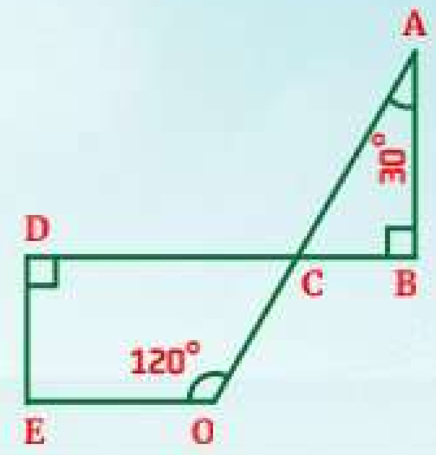


15 In the opposite Figure :



Find the value of x and y .

16 In the opposite Figure :

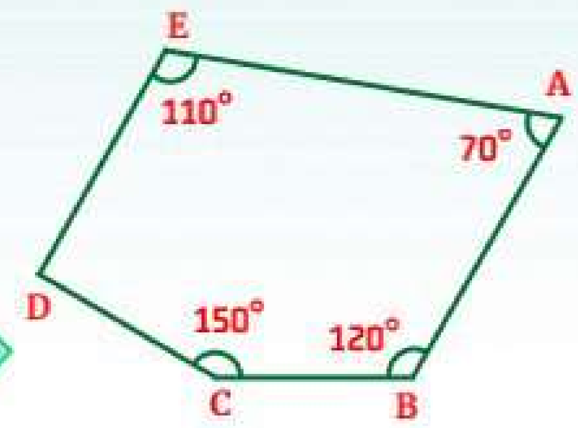


$\overline{AB}$, $\overline{DE}$ are perpendicular to $\overline{BD}$, $\overline{BD} \cap \overline{AO} = \{ C \}$

M (∠ A) = 30 ° , m (∠ O) = 120 °

Find m (∠ E)

17 In the opposite Figure :



ABCDE is a pentagon in which , m (∠ A) = 70° , m (∠ B) 120 °

M (∠ C) = 150 ° , m (∠ E) = 110 °

Find m (∠ D)

Parallelogram

Geometric Shape	Definition	Properties
Parallelogram	It is a quadrilateral , in which its two Opposite sides are parallel.	[1] Each two opposite sides are equal in length and parallel. [2] Each two opposite angles are equal in measure. [3] Each two consecutive angles are supplementary. [4] The two diagonal bisect each other
Rectangle	It is a parallelogram with right angle.	It has the same properties of the parallelogram in addition to : [1] The four angles are right. [2] The two diagonal are equal in measure.
Rhombus	It is a parallelogram in which two adjacent sides are equal in length.	It has the same properties of the parallelogram in addition to : [1] The four sides are equal in length. [2] The two diagonal are perpendicular.
square	It is a rhombus with right angle.	It has the same properties of the parallelogram in addition to : [1] The four sides are equal in length. [2] The four angles are right. [3] The two diagonal are perpendicular and equal in length. And bisect the two angles of the vertices.

School book

Important Exercises

1 A square is .... with a right angle.

2 The quadrilateral in which all side are equal in length is called ....

3 The parallelogram in which diagonals are .... is called a rectangle.

4 ABCD is a parallelogram in which m (∠ A) = 50 ° , then : m (∠ B) = ... °

5 The rectangle is .... with a right angle.

6 The quadrilateral in which diagonals bisect each other is called ....

7 If ABCD is a rhombus , then : .... ⊥ ....

8 The quadrilateral in which two opposite sides are parallel is called ....

9 The quadrilateral is a parallelogram if ....

10 In the parallelogram XYZL , if : m (∠ X) = $\frac{1}{2}$ m (∠ Y) , then : m (∠ Y) = ...

11 The side length of a rhombus whose perimeter 42 cm. equals ....

12 In a parallelogram if the two adjacent sides are equal in length , then the shape is ....

(a) square (b) rhombus (c) rectangle (d) trapezium

13 The two diagonals are equal in length and not perpendicular in ....

(a) parallelogram (b) rhombus (c) rectangle (d) square

14 The parallelogram whose diagonals are perpendicular and not equal in length is called ....

(a) square (b) rhombus (c) rectangle (d) trapezium

15 The diagonals are equal in length and perpendicular in ....

(a) parallelogram (b) rhombus (c) rectangle (d) square

16 The sum of the measures of any two consecutive angles in a parallelogram equals ....

(a) 90 ° (b) 180 ° (c) 270 ° (d) 360 °

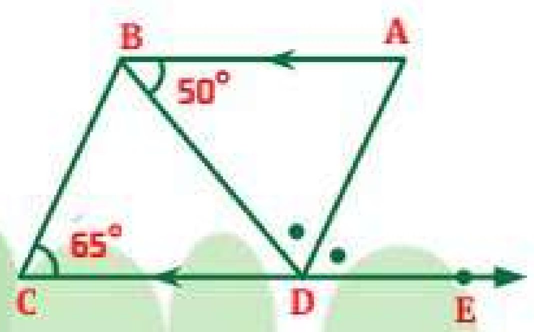
17 In the parallelogram ABCD , if : m (∠ A) + m (∠ C) = 220 ° , then : m (∠ B) = ...

(a) 110 ° (b) 70 ° (c) 55 ° (d) 35 °

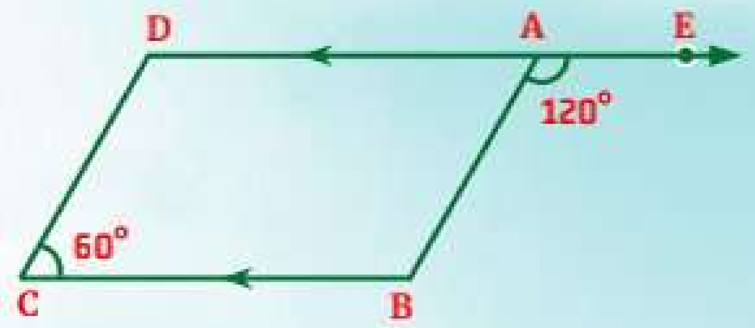
18 **In the opposite Figure :**

$\overrightarrow{DA}$ // $\overrightarrow{BE}$, $\overrightarrow{BA}$ bisects (∠ ADB) , m (∠ ADB) = 50 ° , m (∠ C) = 65°

Prove that ABCD is a parallelogram.



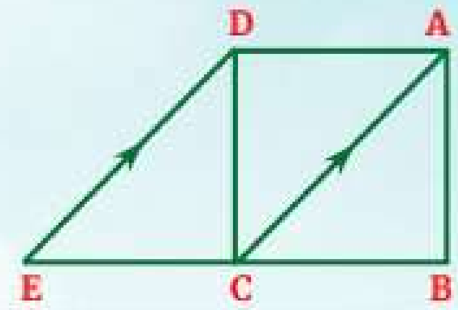
19 In the opposite Figure :



$E \in \overrightarrow{DA}$, $m(\angle EAB) = 120°$, $m(\angle C) = 60°$, $\overrightarrow{DA} \parallel \overline{CB}$

Prove that ABCD is a parallelogram.

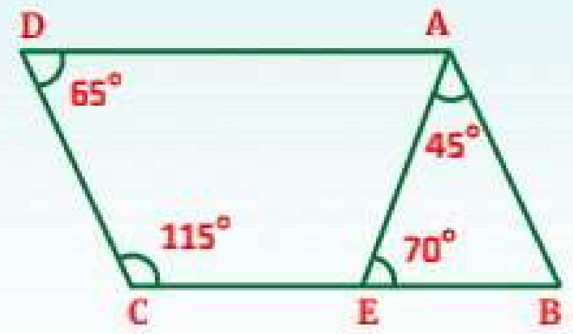
20 In the opposite Figure :



ABCD is a square , $E \in \overrightarrow{DA}$, $\overline{AC} \parallel \overline{DE}$

Prove that ACED is a parallelogram.

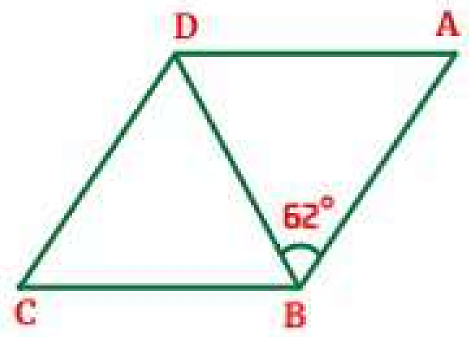
21 In the opposite Figure :



$E \in \overline{BC}$, $m(\angle BAE) = 45°$, $m(\angle AEB) = 70°$, $m(\angle D) = 65°$, $m(\angle C) = 115°$

Prove that ABCD is a parallelogram.

22 In the opposite Figure :



ABCD is a rhombus in which , $\overline{BD}$ is a diagonal and $m(\angle ABD) = 62°$.

Find : $m(\angle A)$

Triangle

1 THEOREM

The sum of the measures of the interior angles of the triangle is 180°

2 THE MEASURE OF THE EXTERIOR ANGLE

The measure of the exterior angle of the triangle is equal to the sum of the measures of its non-adjacent interior angles.

3 REMARKS

1. The measure of the interior angle of the equilateral triangle = 60°
2. The measure of the exterior angle of the equilateral triangle = 120°
3. If the sum of measures of two angles in a triangle is equals 90° , then the third angle is right
4. If the sum of measures of two angles in a triangle is less than 90° , then the third angle is obtuse
5. If the sum of measures of two angles in a triangle is more than 90° , then the third angle is acute
6. If the measure of an angle in a triangle equals the sum of measures of the other two angles , then the triangle is right-angled

4 THEOREM

The ray drawn from the midpoint of a side parallel to another side bisects the third side.

5 COROLLARY

The line segment joining the midpoint of two sides of a triangle is parallel to the third side and Equal to half the length of it.

School book

Important Exercises

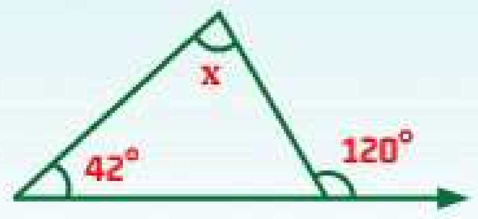
1 The sum of the measures of the interior angles of the triangle equals ....

ⓐ 90° ⓑ 180° ⓒ 270° ⓓ 360°

2 The measure of the exterior angle of the equilateral triangle is ....

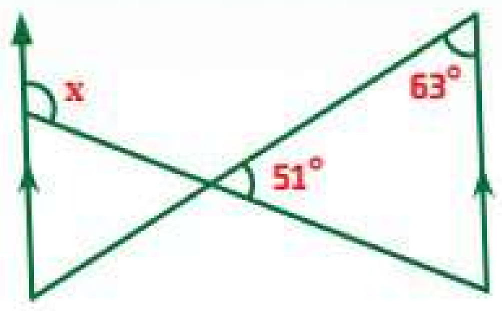
ⓐ 30° ⓑ 45° ⓒ 60° ⓓ 120°

3 The measure of the exterior angle of the equilateral triangle is ....



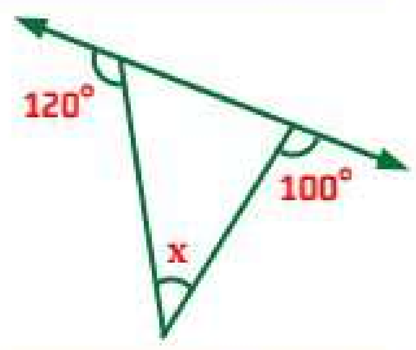
ⓐ 42° ⓑ 78°
ⓒ 120° ⓓ 18°

4 The measure of the exterior angle of the equilateral triangle is ....



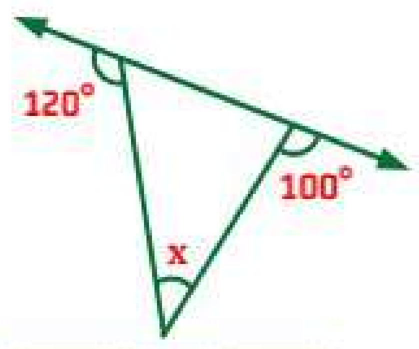
ⓐ 51° ⓑ 63°
ⓒ 114° ⓓ 66°

5 The measure of the exterior angle of the equilateral triangle is ....



ⓐ 120° ⓑ 60°
ⓒ 240° ⓓ 40°

6 The measure of the exterior angle of the equilateral triangle is ....



ⓐ 120° ⓑ 60°
ⓒ 240° ⓓ 40°

7 ABC is a triangle in which : m (∠ B) = 50° , m (∠ A) = 60° , then : m (∠ C) = ...

ⓐ 50° ⓑ 60° ⓒ 70° ⓓ 80°

8 ABC is a triangle in which : m (∠ A) = m (∠ B) = 2 m (∠ C) , then : m (∠ C) = ...

ⓐ 36° ⓑ 72° ⓒ 108° ⓓ 144°

9 ABC is a triangle in which : m (∠ A) > m (∠ B) + m (∠ C) , then : (∠ A) is ....

ⓐ an acute ⓑ a right ⓒ an obtuse ⓓ a straight

10 ABC is a triangle in which : m (∠ A) = m (∠ B) + m (∠ C) , then : (∠ A) is ....

ⓐ an acute ⓑ a right ⓒ an obtuse ⓓ a straight

11 ABC is a triangle in which : 9 m (∠ A) = m (∠ B) + m (∠ C) , then : m (∠ A) = ...

ⓐ 18° ⓑ 36° ⓒ 54° ⓓ 72°

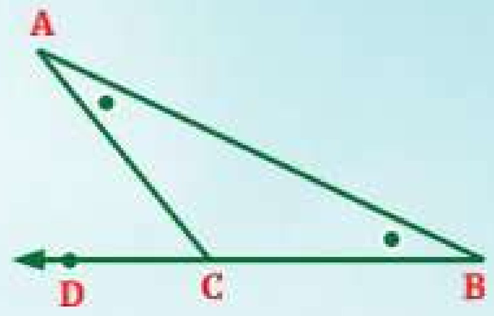
12 any triangle has at least two .... interior angles

ⓐ acute ⓑ right ⓒ obtuse ⓓ straight

13 The line segment joining the midpoint of two sides of a triangle equals ....

14 If the measure of an angle in a triangle equals the sum of measures of the other two angles , then the triangle is ....

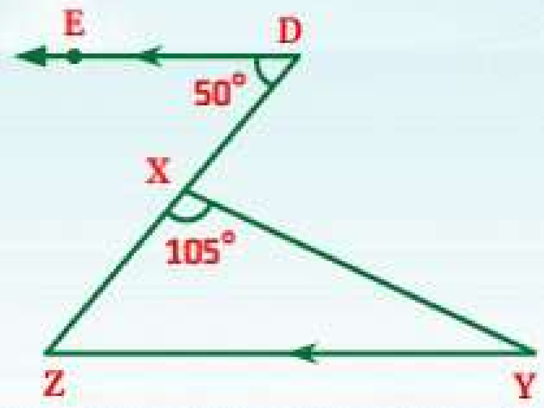
15 **In the opposite Figure :**



m (∠ A) = m (∠ B) = 25 °.

Find m (∠ ACD)

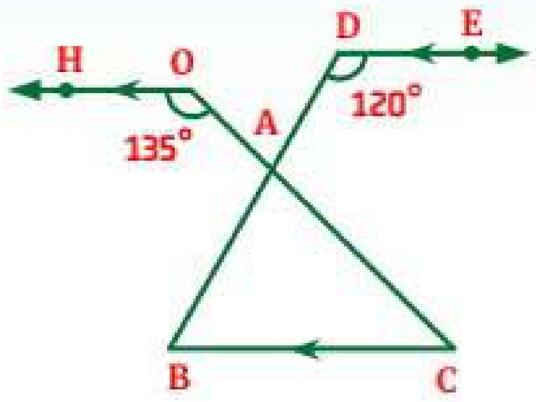
16 **In the opposite Figure :**



$\overrightarrow{DE}$ // $\overline{YZ}$, m (∠ ZDE) = 50 ° , m (∠ YXZ) = 105 °.

Find m (∠ Z) , m (∠ Y) , m (∠ YXD)

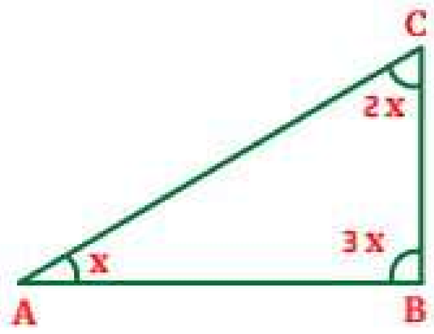
17 **In the opposite Figure :**



$\overrightarrow{DE}$ // $\overrightarrow{OH}$ // $\overline{BC}$, m (∠ ADE) = 120 ° , m (∠ AOH) = 135 °.

Calculate the measures of the angles of the triangle ABC.

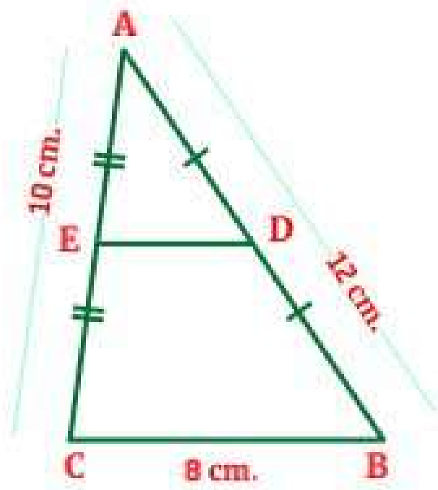
18 **In the opposite Figure :**



ABC is a triangle in which : m (∠ A) = x , m (∠ B) = 3 x , m (∠ C) = 2 x.

Calculate the measures of the angles of the triangle ABC.

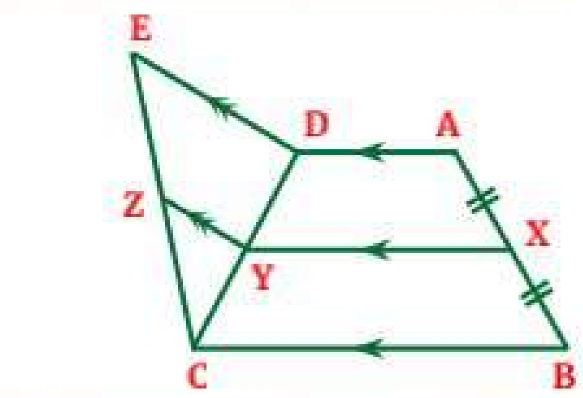
19 **In the opposite Figure :**



ABC is a triangle in which : AB = 12 cm. , BC = 10 cm.

Find the perimeter of Δ ABC.

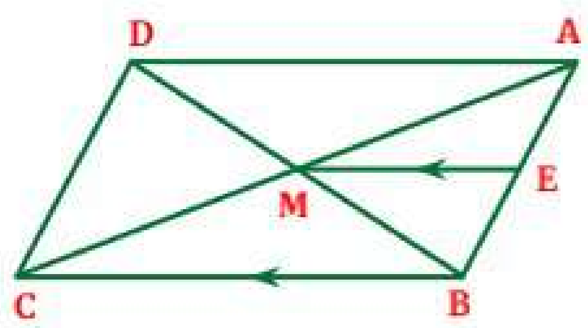
20 **In the opposite Figure :**



X is the midpoint of $\overline{AB}$, Y ∈ $\overline{CD}$, Z ∈ $\overline{CE}$, $\overline{AD}$ // $\overline{XY}$ // $\overline{BC}$, $\overline{YZ}$ // $\overline{DE}$

Prove that : CZ = ZE

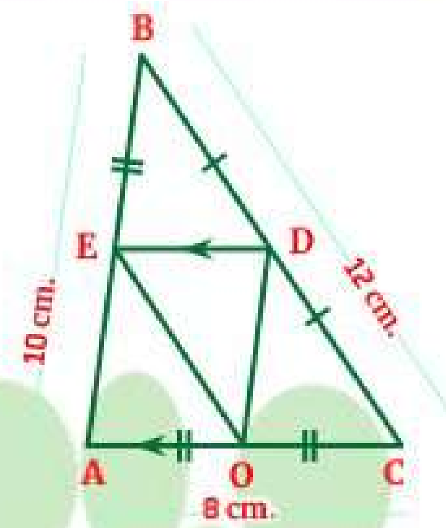
21 **In the opposite Figure :**



ABCD is a parallelogram , M is the intersection of its diagonals , $\overrightarrow{ME}$ // $\overline{CB}$

Prove that : AE = EB

22 **In the opposite Figure :**



ABC is a triangle in which D is the midpoint of $\overline{AB}$, O is the midpoint of $\overline{CB}$, E ∈ $\overline{AC}$ such that $\overline{DE}$ // $\overline{BC}$, AB = 10 cm. , BC = 12 cm. , AC = 8 cm.

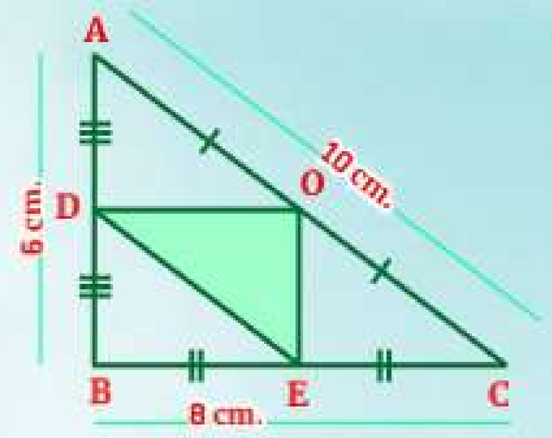
Prove that : DBOA is a parallelogram and find the perimeter of Δ EDO.

23 In the opposite Figure :

D , E , O are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ respectively , AB = 6 cm.

BC = 8 cm. , AC = 10 cm.

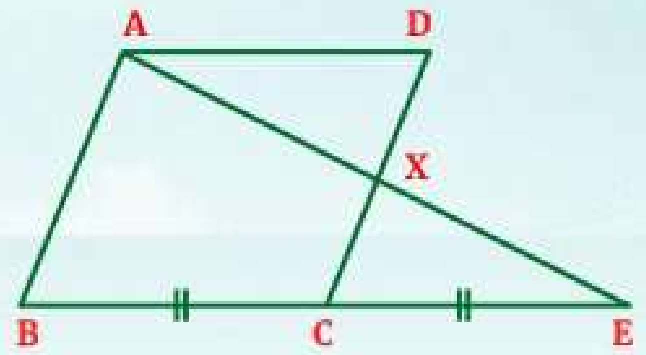
Find : the perimeter of △ DEO.



24 In the opposite Figure :

ABCD is a parallelogram , E ∈ $\overrightarrow{BC}$ such that CE = BC , $\overline{AE}$ ∩ $\overline{DC}$ = { X }

Prove that : AX = XE

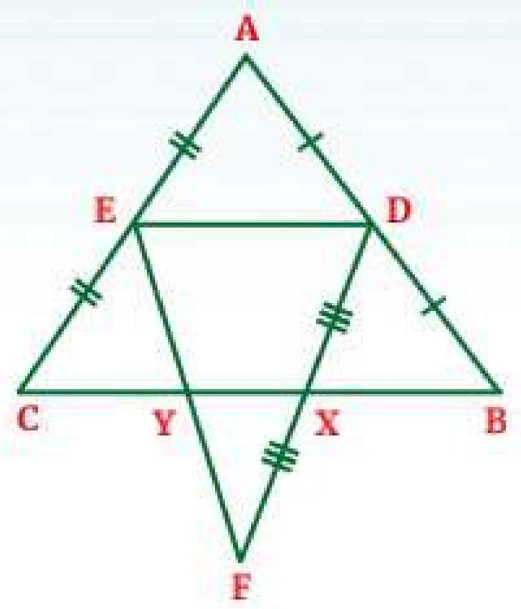


25 In the opposite Figure :

ABC is a triangle in which D , E are the midpoints of $\overline{AB}$, $\overline{AC}$

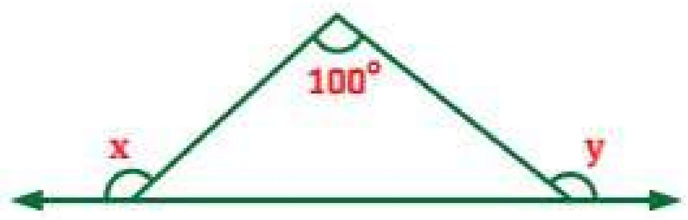
Respectively , CB = 12 cm. , and DX = XF

Find : the length of $\overline{XY}$



26 x + y = ...

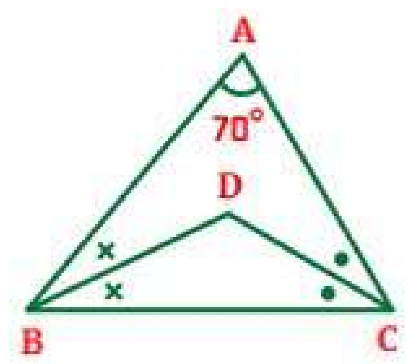
- (a) 100 °
- (b) 80 °
- (c) 260 °
- (d) 280 °



27 In the opposite Figure :

If : $\overline{BD}$ bisects (∠ ABC) , $\overline{CD}$ bisects (∠ ACB) , then : m (∠ D) = ...

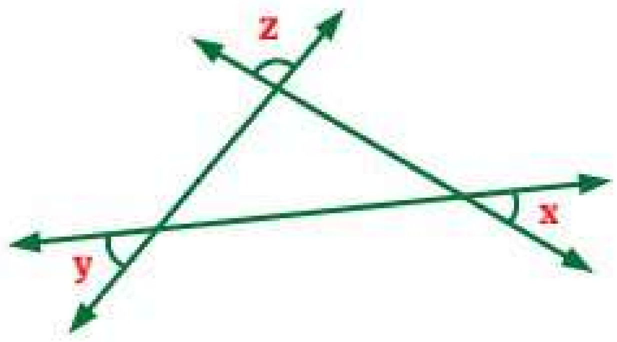
- (a) 70 °
- (b) 140 °
- (c) 110 °
- (d) 120 °



28 In the opposite Figure :

x + y + z = ...

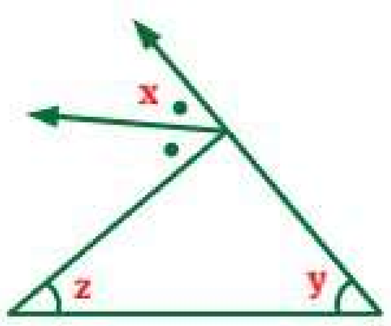
- (a) 180 °
- (b) 90 °
- (c) 360 °
- (d) 270 °



28 In the opposite Figure :

If : z complements y , then : x = ...

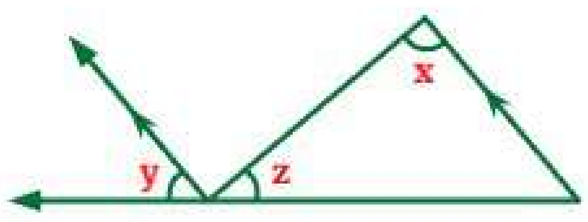
- (a) 90 °
- (b) 45 °
- (c) 30 °
- (d) 60 °



28 In the opposite Figure :

x + y + z = ...

- (a) 90 °
- (b) 180 °
- (c) 270 °
- (d) 360 °



Pythagoras

THE THEOREM

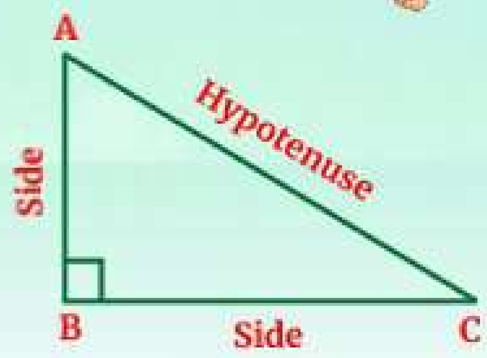
In the right-angled triangle , area of the square drawn on the hypotenuse equals sum Of the areas of the squares drawn on the other two sides.

EXPLANATION

$\because$ ABC is a right angled triangle at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore (BC)^2 = (AC)^2 - (AB)^2$ and $(AB)^2 = (AC)^2 - (BC)^2$



School book

Important Exercises

1 ABC is a right-angled triangle at B in which : AB = 6 cm. , BC = 8 cm. , then : AC = ...

ⓐ 14 cm. ⓑ 10 cm. ⓒ 8 cm. ⓓ 6 cm.

2 XYZ is a right-angled triangle at X in which : XY = 12 cm. , YZ = 13 cm. , then : XZ = ...

ⓐ 4 cm. ⓑ 5 cm. ⓒ 6 cm. ⓓ 7 cm.

3 ABC is a triangle in which : $(AC)^2 = (AB)^2 + (BC)^2$, AC = 25 cm. , BC = 24 cm. , then : AB = ...

ⓐ 5 cm. ⓑ 6 cm. ⓒ 7 cm. ⓓ 8 cm.

4 ABC is a triangle in which : $(AC)^2 = (AB)^2 + (BC)^2$, AC = 25 cm. , BC = 24 cm. , then : AB = ...

ⓐ 5 cm. ⓑ 6 cm. ⓒ 7 cm. ⓓ 8 cm.

5 ABCD is a rectangle of two dimensions 20 cm. , 15 cm. , then the length of its diagonal = ...

ⓐ 35 cm. ⓑ 70 cm. ⓒ 30 cm. ⓓ 25 cm.

6 If the lengths of the two diagonals of a rhombus are 12 cm. and 16 cm. , then its side length = ...

ⓐ 20 cm. ⓑ 10 cm. ⓒ 6 cm. ⓓ 8 cm.

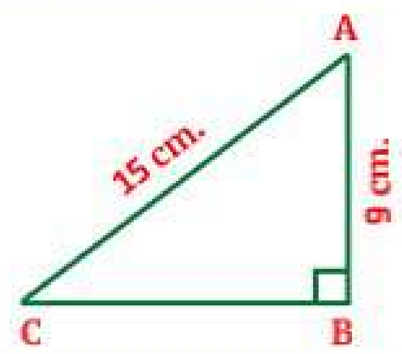
7 If the lengths of the two diagonals of a rhombus are 6 cm. and 8 cm. , then its perimeter = ...

ⓐ 5 cm. ⓑ 10 cm. ⓒ 15 cm. ⓓ 20 cm.

8 **In the opposite Figure :**

ABC is a right-angled triangle at B in which : AB = 9 cm. , AC = 15 cm. , then : BC = ...

ⓐ 13 cm. ⓑ 12 cm. ⓒ 11 cm. ⓓ 10 cm.

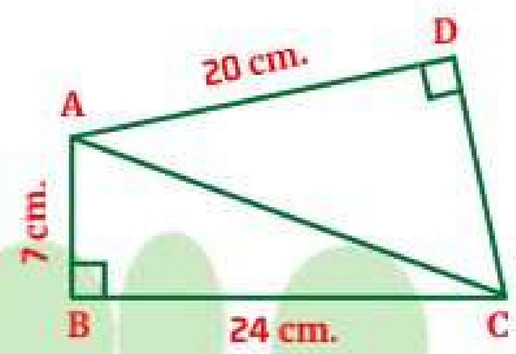


9 **In the opposite Figure :**

ABC is a right-angled triangle at B , $\overline{AD} \perp \overline{DC}$

, AD = 20 cm. , AB = 7 cm. , BC = 24 cm.

Find : the length of each $\overline{AC}$, $\overline{CD}$.

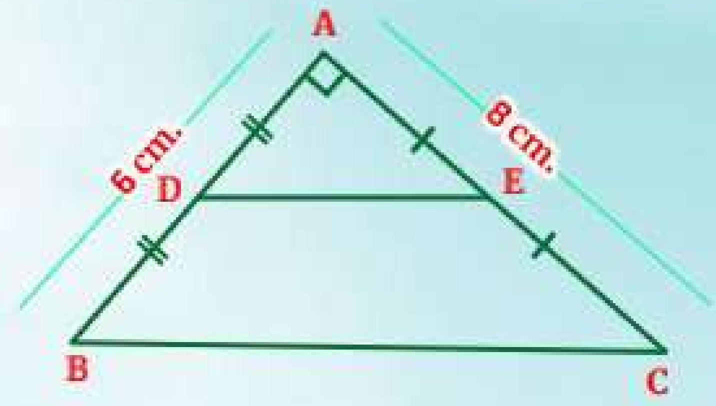


10 **In the opposite Figure :**

ABC is a right-angled triangle at A , AB = 6 cm. , AC = 8 cm.

, D , E are the midpoints of $\overline{AB}$, $\overline{AC}$

Find : the length of $\overline{DE}$

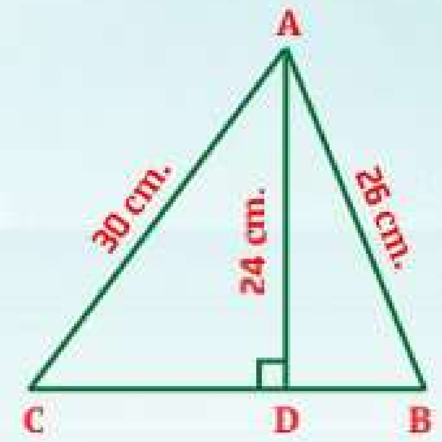


11 **In the opposite Figure :**

$\overline{AD} \perp \overline{BC}$, If AD = 24 cm. , AB = 26 cm. , AC = 30 cm.

, D ∈ $\overline{BC}$ where AD = 10 cm. , AC = 8 cm.

Find : the length of $\overline{BC}$ and the area of the triangle ABC

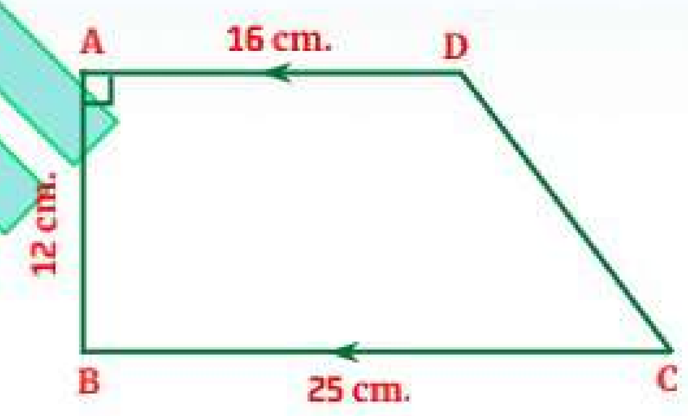


12 **In the opposite Figure :**

ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$, m (∠ A) = 90 °

, AB = 12 cm. , BC = 25 cm. , AD = 16 cm.

Find : the length of $\overline{DC}$.

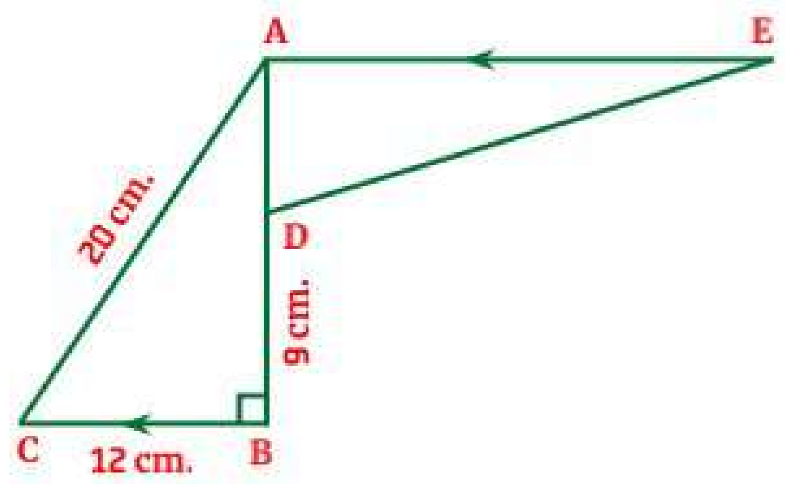


13 **In the opposite Figure :**

Δ ABC in which : m (∠ B) = 90 ° , $\overline{AE}$ // $\overline{BC}$, If BC = 12 cm.

, AC = 20 cm. , D ∈ $\overline{AB}$ where BD = 9 cm. AE = 2 BC

Find : the length of each $\overline{AD}$, $\overline{ED}$.



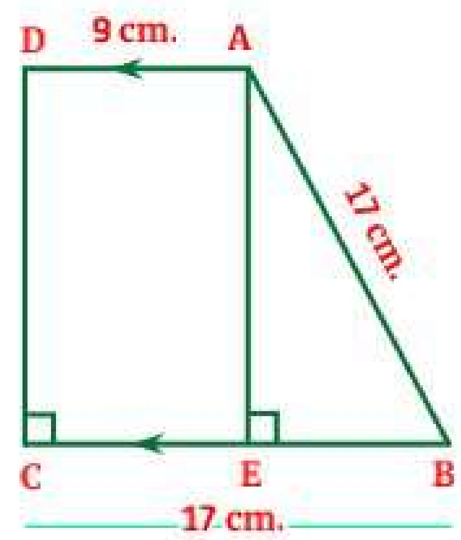
14 **In the opposite Figure :**

ABCD is a trapezium , $\overline{AD}$ // $\overline{BC}$, m (∠ DCB) = 90 ° , $\overline{AE} \perp \overline{BC}$

If AB = BC = 17 cm. and AD = 9 cm.

Find : the length of each $\overline{DC}$, $\overline{ED}$.

and **calculate** the area of the trapezium ABCD

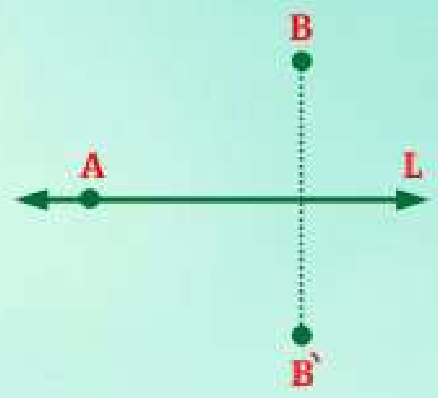


15 **Find :** the length of the diagonal of a rectangle whose area 48 $cm.^2$ and of width 6 cm.

Reflection

The image of the point A by reflection in the straight line L where $A \in L$ is A

The image of the point B by reflection in the straight line L where $B \notin L$ is B`



REFLECTION IN THE CARTESIAN COORDINATES

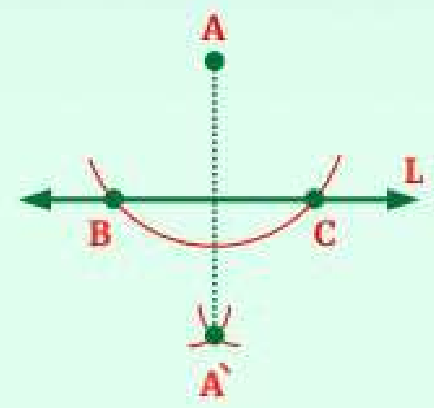
The image of the point A (x , y) by reflection in the (X-axis) is A` (x , – y)

The image of the point A (x , y) by reflection in the (Y-axis) is A` (– x , y)

The image of the point A (x , y) by reflection in the (Origin point) is A` (– x , – y)

REFLECTION OF A POINT IN A GIVEN STRAIGHT LINE

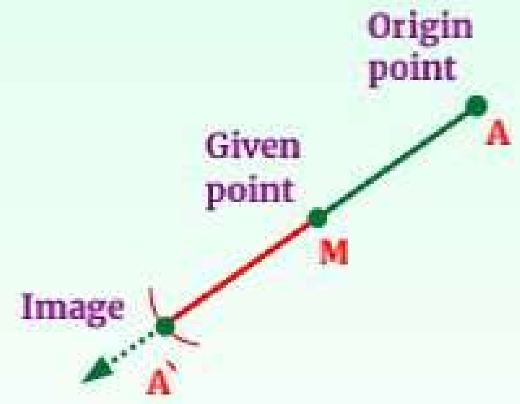
1. By using the compass we draw an arc with Centre A to cut L at B and C
2. With the same radius length we draw two arcs with centres B and C in the other side of L to intersect at A`



REFLECTION OF A POINT IN A GIVEN POINT

To find the image of by reflection in the point M :

1. Draw $\overrightarrow{AM}$
2. With the radius length MA , drew an arc with Centre M in the other side of M intersects $\overrightarrow{AM}$ at a point A`



School book

Important Exercises

1 The image of the point (2 , – 5) by reflection in X-axis is ....

ⓐ (2 , – 5) ⓑ (2 , 5) ⓒ (– 2 , – 5) ⓓ (5 , 2)

2 The image of the point (3 , – 5) by reflection in Y-axis is ....

ⓐ (3 , 5) ⓑ (– 3 , – 5) ⓒ (– 3 , 5) ⓓ (– 5 , 3)

3 The point (– 3 , 2) is the image of the point (3 , 2) by reflection in ....

ⓐ X-axis ⓑ Y-axis ⓒ Origin point ⓓ the point M

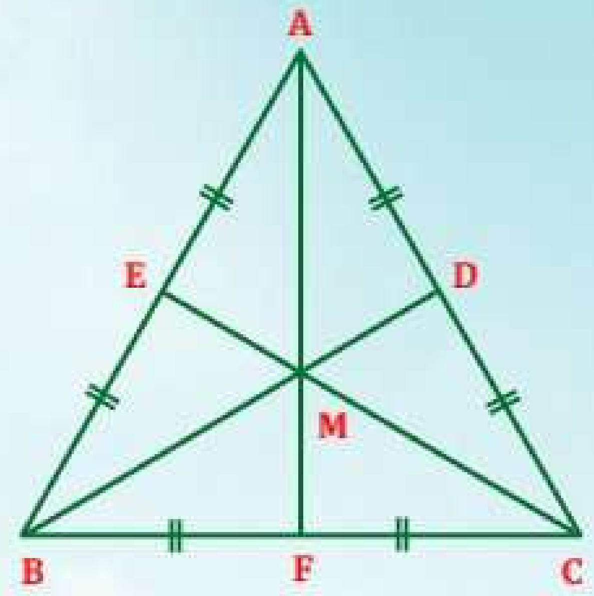
4 The image of the point (– 2 , 3) by reflection in the origin point is ....

ⓐ (2 , 3) ⓑ (– 2 , – 3) ⓒ (2 , – 3) ⓓ (– 2 , 3)

5 If The image of the point A by reflection in the origin point is (3 , – 5) , then A is ....

ⓐ (3 , 5) ⓑ (– 3 , – 5) ⓒ (3 , – 5) ⓓ (– 3 , 5)

6 **In the opposite Figure :**



ABC is an equilateral triangle where D , E , and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively and $\overline{AB} \cap \overline{BC} \cap \overline{AC} = \{ M \}$, **Complete :**

1. Axes of symmetry of △ ABC are .... , ... , ...
2. $\overline{AB}$ is the reflected image of $\overline{AC}$ by reflection in ....
3. The reflected image of $\overline{AF}$ by reflection in $\overleftrightarrow{BF}$ is ....
4. The reflected image of $\overline{CF}$ by reflection in $\overleftrightarrow{AE}$ is ....
5. The reflected image of △ AMD by reflection in $\overleftrightarrow{AE}$ is ....
6. The reflected image of △ AMB by reflection in $\overleftrightarrow{AE}$ is ....
7. The reflected image of △ BMC by reflection in $\overleftrightarrow{CD}$ is ....

7 **Find** the image of the triangle ABC where A (– 6 , – 1) , B (– 2 , – 1) and C (– 5 , – 6) by reflection in X–axis.

8 **Find** the image of the triangle ABC where A (2 , – 3) , B (– 1 , 2) and C (3 , 1) by reflection in Y–axis.

9 **Draw** the image of the rectangle ABCD where A (2 , 2) , B (– 3 , 2) with width 3 units by reflection In X–axis . **How many** cases can you draw.

10 **Find** the image of the triangle ABC where A (– 2 , 4) , B (5 , 0) and C (3 , – 3) by reflection in origin point.

11 **Draw** the triangle ABC where AB = 3 cm. , BC = 4 cm. , m (∠ ABC) = 90 °.

Then **draw** its image by reflection in straight line $\overleftrightarrow{BC}$

Translation

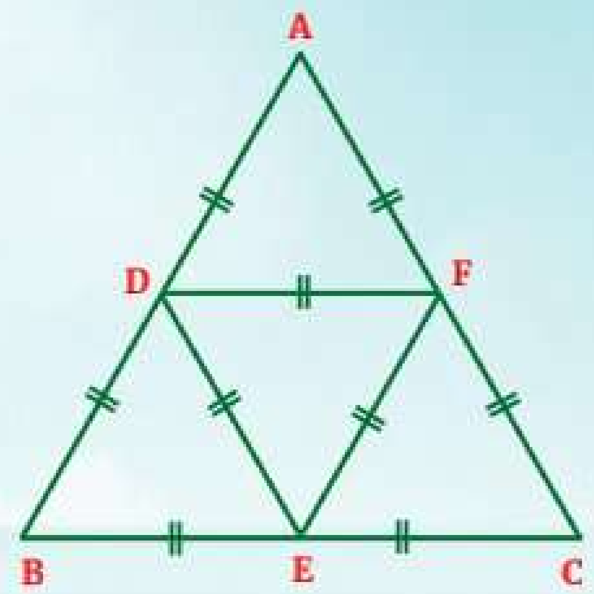
The image of the point A (x , y) by translation (a , b) is (x + a , y + b)

If M (2 , 3) , N (1 , 4) Then the translation of any point (x , y) with magnitude MN in the direction $\overrightarrow{MN}$ is Equivalent to the translation (1 – 2 , 4 – 3) = (– 1 , 1) or (x – 1 , y + 1)

School book

Important Exercises

1 In the opposite Figure : Complete :



1. The image of Δ ADF by a translation of magnitude AD in the direction of $\overrightarrow{AD}$ is ....
2. Δ FEC is the image of Δ DBE by a translation of magnitude .... in the direction of ....
3. The image of Δ FCE by a translation of magnitude FD in the direction of $\overrightarrow{FD}$ is ....

2 The image of the point (– 1 , 3) by translation in the origin point is (4 , – 2) is ....

ⓐ (3 , 1) ⓑ (3 , – 1) ⓒ (5 , 1) ⓓ (5 , – 5)

3 The image of the point (5 , 3) by translation (x , y) ⟹ (x + 3 , y – 1) is ....

4 The image of the point (2 , 3) by translation MN in the direction $\overrightarrow{MN}$ where M (2 , – 1) , N (5 , 1) is ....

5 The image of the point (5 , – 3) by translation 3 units in negative direction of X-axis is ....

6 The image of the point (3 , 5) by translation 4 units in negative direction of Y-axis is ....

7 The image of the point (– 4 , – 3) by translation 3 units in positive direction of X-axis is ....

8 If : A` (3 , 4) is the image of A by translation (x – 2 , y – 3) ,Then : the point A is ....

ⓐ (1 , 1) ⓑ (1 , 7) ⓒ (5 , 1) ⓓ (5 , 7)

9 Draw the square ABCD where A (1 , 1) , B (4 , 2) , C (3 , 5) and D (0 , 4).

Graph the image of the square by translation AB in the direction of $\overrightarrow{AB}$

10 Draw $\overline{AB}$ where A (4 , 3) , B (– 1 , 1), then Find the image of $\overline{AB}$ by translation (x + 2 , y – 1).

11 Using the square lattice , Draw triangle ABC where A (2 , 4) , B (4 , 2) , C (3 , 1) then Find the Then map its image of the square by translation (x + 2 , y + 3)

12 Draw the triangle ABC where A (6 , 1) , B (1 , 1) , C (3 , 5).

Graph the image of the triangle by translation CB in the direction of $\overrightarrow{CB}$

Rotation

POSITIVE AND NEGATIVE MEASURE

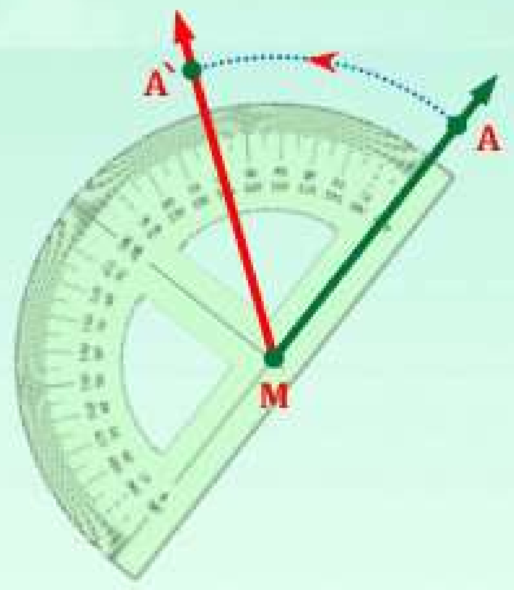
If the rotation is anticlockwise , then the measure of the rotation angle is positive

, If the rotation is clockwise , then the measure of the rotation angle is negative



FINDING THE IMAGE OF A POINT BY ROTATION AROUND A POINT

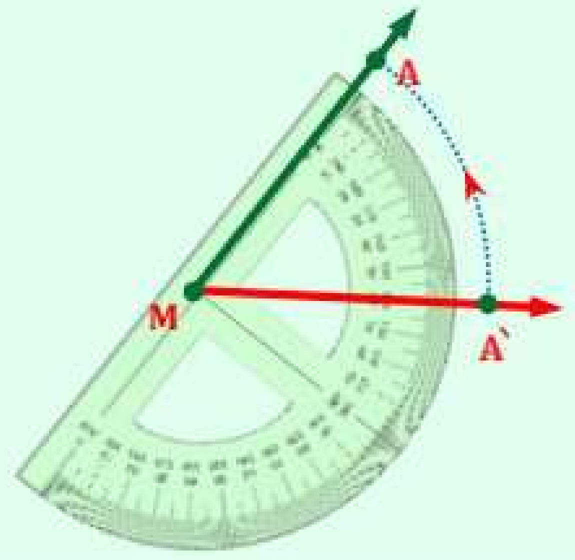
Draw the image of the point A by rotation (M , 60 °)



1. Draw the ray $\overrightarrow{MA}$
2. Put the protractor with its straight edge on $\overrightarrow{MA}$ and in the anticlockwise direction , then draw $\overline{MC}$ such that m (∠ AMC) = 60 °
3. With the radius length MA , drew an arc with Centre M intersects $\overrightarrow{MC}$ at a point A`

FINDING THE IMAGE OF A POINT BY ROTATION AROUND A POINT

Draw the image of the point A by rotation (M , – 60 °)



1. Draw the ray $\overrightarrow{MA}$
2. Put the protractor with its straight edge on $\overrightarrow{MA}$ and in the anticlockwise direction , then draw $\overline{MC}$ such that m (∠ AMC) = 60 °
3. With the radius length MA , drew an arc with Centre M intersects $\overrightarrow{MC}$ at a point A`

ROTATION IN THE CARTESIAN PLAN

1. The image of the point A (x , y) by rotation (O , 90 °) or (O , – 270 °) is (– y , x)
2. The image of the point A (x , y) by rotation (O , – 90 °) or (O , 270 °) is (y , – x)
3. The image of the point A (x , y) by rotation (O , – 180 °) or (O , 180 °) is (– x , – y)
4. The image of the point A (x , y) by rotation (O , – 360 °) or (O , 360 °) is (x , y)

School book

Important Exercises

1 The image of the point (– 3 , 5) by Rotation about the origin point by an angle of measure 90 ° is ....

ⓐ (5 , 3)　ⓑ (– 5 , 3)　ⓒ (3 , 5)　ⓓ (– 5 , – 3)

2 The image of the point (2 , – 1) by Rotation around the origin point by an angle of measure 180 ° is ....

ⓐ (1 , 2)　ⓑ (2 , – 1)　ⓒ (– 2 , 1)　ⓓ (2 , – 1)

3 If the image of the point (– 4 , 0) by Rotation about the origin point is (0 , – 4).
Then the measure of the rotation is ....

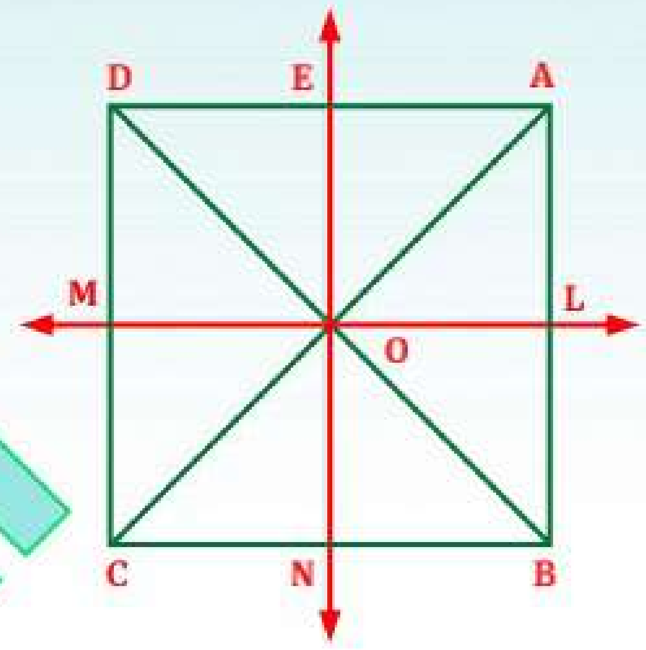
ⓐ 90° ⓑ 180° ⓒ 270° ⓓ 360°

4 If the image of the point (5 , – 3) by Rotation about the origin point is itself.
Then the measure of the rotation is ....

ⓐ 90° ⓑ 180° ⓒ 270° ⓓ 360°

5 **In the opposite Figure :**

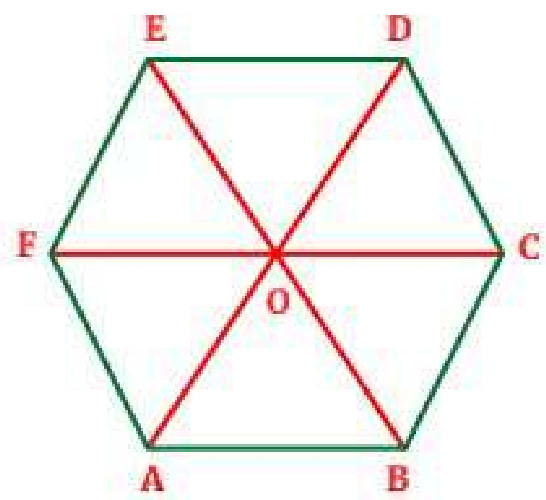
ABCD is a square of side length 6 cm. and the origin point its center **complete :**



1. The image of $\triangle$ AOL by a translation 3 cm. in the direction of $\overrightarrow{AB}$ is ....
2. The image of $\triangle$ AOL by reflection in $\overleftrightarrow{EN}$ is ....
3. The image of $\triangle$ AOL by rotation about O with an angle of measure – 90° is ....

6 **In the opposite Figure :**

ABCDEF is a regular hexagon , **complete :**



1. The image of $\triangle$ EFO by a translation DC in the direction of $\overrightarrow{DC}$ is ....
2. The image of $\triangle$ DCO by a translation EF in the direction of $\overrightarrow{EF}$ is ....
3. The image of $\triangle$ EDO by rotation about O with an angle of measure 60° is ....
4. The image of $\triangle$ COB by rotation about O with an angle of measure – 120° is ....
5. The image of $\triangle$ BOC by reflection in $\overleftrightarrow{DA}$ is ....
6. The image of $\triangle$ DCO by reflection in $\overleftrightarrow{FC}$ is ....

7 **Draw** the triangle ABC where AB = 5 cm. , BC = 12 cm. , m (∠ B) = 90°.

1. **Draw** X , the image of B by translation 9 cm. in the direction of $\overrightarrow{BA}$
2. **Draw** Y , the image of B by rotation (A , – 90°)

8 **Draw** the equilateral triangle ABC with side length 6 cm. then **Draw** the image of the triangle ABC by rotation R (A , 60°)

9 **Draw** the rectangle ABCD with vertices A (0 , 0) , B (0 , 2) , C (4 , 2) and D (4 , 0).
then **Draw** its image by rotation about O with an angle of measure :

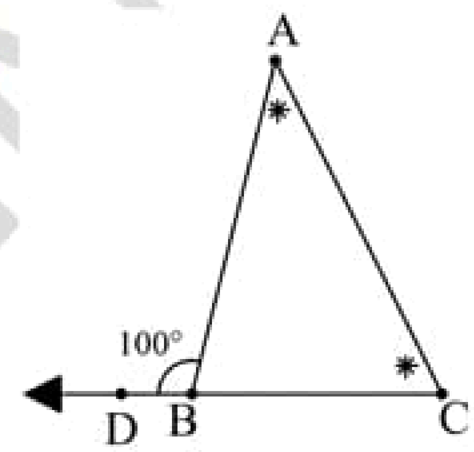
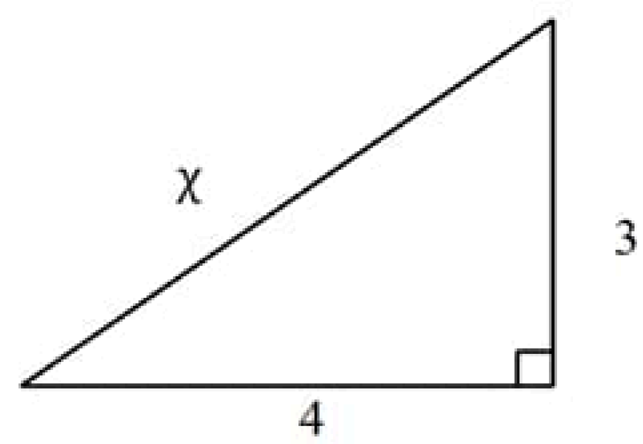
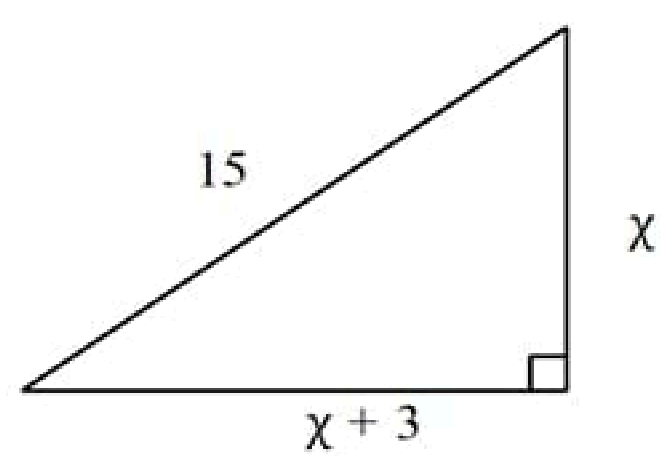
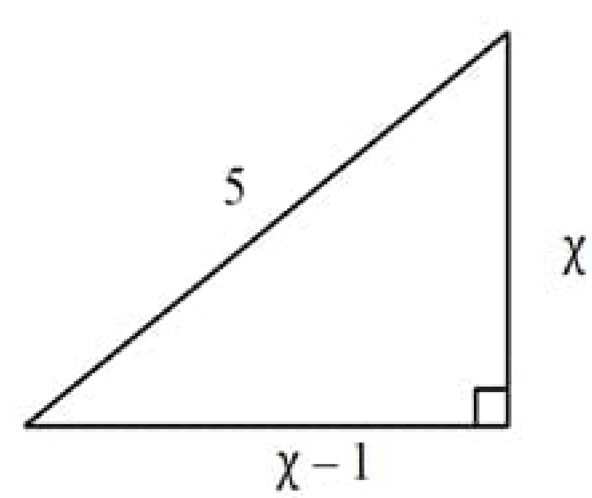
1. 90° 2. 180° 3. 270° 4. 360°

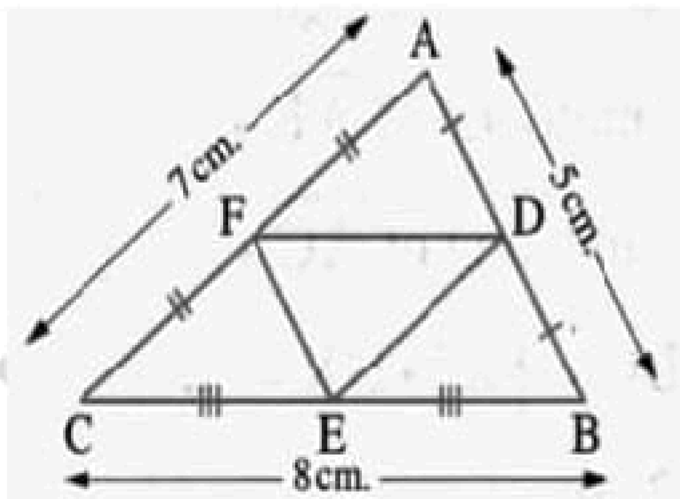
Best wishes, Mr. Abdelrahman Essam

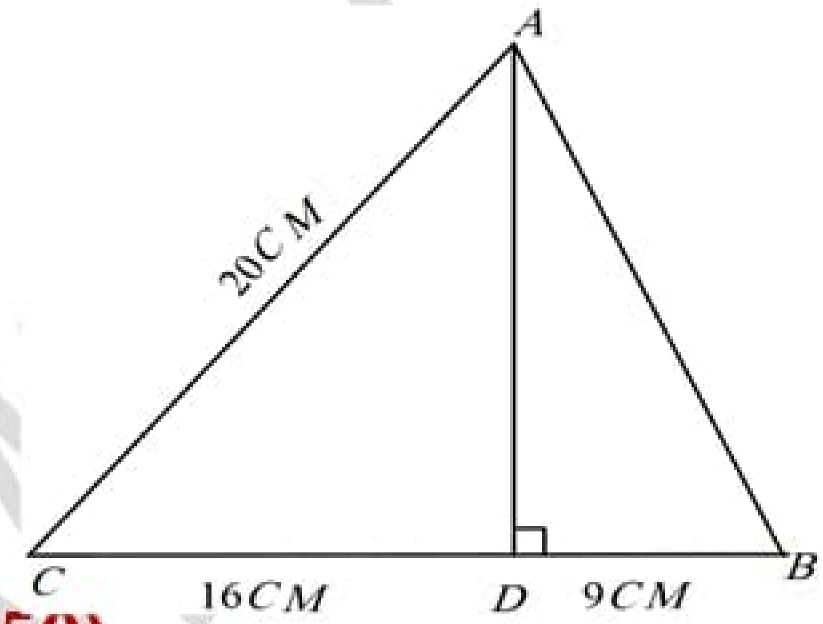
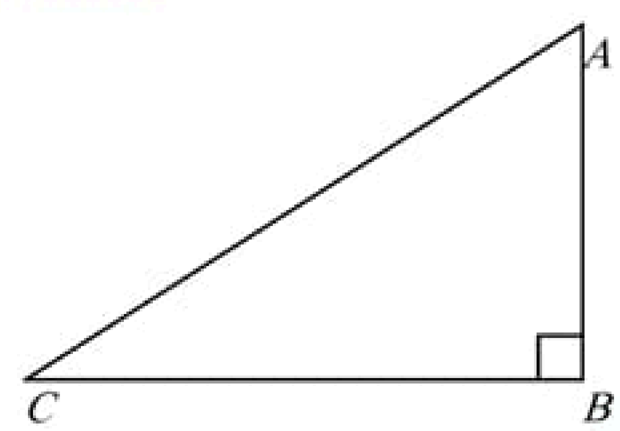
Answer the following questions:

Choose the correct answer from those given:

1.	The measure of the exterior angle at any vertex of the equilateral triangle = ... (60° or 120° or 150° or 30°)
2.	If À (4, −5) is the image of A by translation $(x, y) \to (x - 2, y + 1)$ then the point A is .... ((6, −4) or (4, −4) or (2, −4) or (6, −6)
3.	Any triangle has at least two .... angles. (acute or obtuse or right or reflex)
4.	If the measures of two angles in a triangle are 30° and 50°, then the triangle is .... (acute-angled or right-angled) (obtuse-angled or equilateral)
5.	If Δ ABC is right-angled at B, AB = 20 cm. and AC = 25 cm., then BC= .... cm. (225 or 400 or 15 or 10)
6.	In Δ ABC: if m (∠ B) = m (∠ A) + m (∠ C), then ∠ B is .... (acute. or right. or obtuse. or reflex.)
7.	Δ XYZ is right-angled at Y, XY = 12 cm., YZ = 5cm., Then XZ = ...cm (169 or 25 or 17 or 13)
8.	Δ ABC is right-angled at B, if the measure of the exterior angle at A is 120°, then m (∠ C) =............ (60° or 90° or 120° or 30°)

9.	The length of the line segment joining the midpoints of two sides of a triangle is equal to......... the length of the third side. (twice or half or third or quarter)
10.	In Δ ABC, if m (∠ A) = m (∠ C) – m (∠ B), m (∠ B) = 50°, then m (∠ A) =............. (40° or 90° or 50° or 45°)
11.	D∈ $\overrightarrow{CB}$, m (∠ ABD) = 100º, m (∠A) = m (∠ C), then m (∠ C) = ... (40º or 80° or 50° or 100º) 
12.	Which of the following relations is true? $x = 4^2 + 3^2$ or $x^2 = 4^2 - 3^3$ $x^2 + 9 = 16$ or $x^2 = 25$ 
13.	Which of the following relations is true? $x + 3 + x = 15$ $x^2 + 3x = 108$ $(x + 3)^2 = 15 - x^2$ $x^2 + 6x + 9 = 225$ 
14.	Which of the following relations is true? $x + (x - 1) = 25$ $(x - 1)^2 - x^2 = 25$ $x^2 + (x - 1)^2 = 5$ $(x^2 - x = 12$ 

15.	If ABCD is a square, then $(AC)^2$ = ... (AB or $(AB)^2$ or $2(AB)^2$ or $4\ (AB)^2$
16.	AB = 5 cm., BC = 8 cm., AC = 7 cm., D, E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively. The perimeter of: Δ DEF  (15 or 10 or 20)
17.	The image of the point (−2, 3) by translation of magnitude of 4 units in the negative direction of the y-axis is .... ((2, 3) or (−2, 7) or (−6, 3) or (−2, −1)
18.	The sum of measures of the interior angles of a triangle equals the measure of .... angle. (a right or a straight or an acute or a reflex)
19.	In Δ XYZ, if: m (∠ X) = 50°, m (∠ Y) = 100°, then: m (∠ Z) = ... (30° or 50° or 80° or 100°)
20.	In Δ ABC, if: m (∠A) + m (∠B) = 110°, then: m (∠C) = ... (110° or 90° or 70° or 55°)
21.	If the measures of two angles in a triangle are 35° and 45°, then the triangle is .... (acute-angled or right-angled) (obtuse-angled or equilateral)
22.	The measure of the exterior angle of the equilateral triangle at any one of its vertices equals .... (60° or 120° or 150° or 30°)

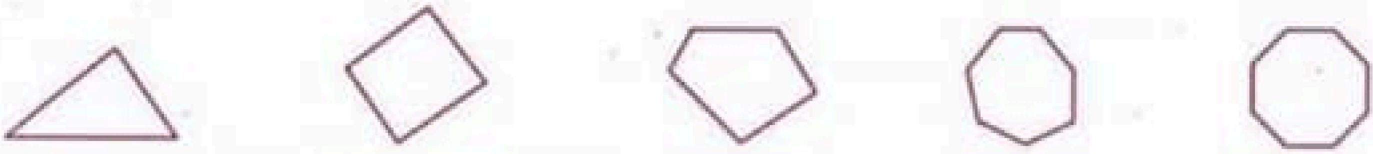
23.	In Δ ABC, if: m (∠*B*) > m (∠A) + m< (∠C), then ∠A is …….. (acute or right or obtuse or reflex)
24.	In the opposite figure $\overline{AD} \perp \overline{BC}$, BD = 9cm DC = 16cm, and AC = 20cm then AD = (5 or 12 or 15) AB = (5 or 12 or 15) Area of Δ ABC = (15 or 120 or 150) 
25.	If the measures of an angles in a triangle equals the sum of measures of the other two angles, then the triangle is …….. (acute or right or obtuse)
26.	If the measures of an angles greater than the sum of measures of the other two angles, then then the triangle is ……… (acute or right or obtuse)
27.	If Δ ABC is right-angled at B then $(AC)^2$ = ……. ($(AC)^2$- $(BC)^2$) ($(AC)^2$ – $(AB)^2$) ($(AB)^2$ + $(BC)^2$) 

28.	The number of axes of symmetry of
a.	The equilateral triangle is .... (2 or 3 or 4 or 6)
b.	The isosceles triangle is .... (2 or 3 or 1 or 0)
c.	The scalene triangle is .... (2 or 0 or 3 or 4)
d.	The parallelogram is .... (0 or 2 or 3 or 4)
e.	The rectangle is .... (0 or 2 or 3 or 4)
f.	The rhombus is ...... (2 or 3 or 4 or 5)
g.	The square is ...... (2 or 4 or 6 or 3)
h.	The trapezium which is not isosceles is .... (2 or 0 or 1 or 4)
i.	The isosceles trapezium is .... (2 or 0 or 1 or 4)
j.	The circle is .... (0 or 5 or 4 or in finite)
29.	The image of the point (5,0) in the .... it self (y – axis or x – axis or origin)
30.	The image of the point (-5, 0) by reflection in the .... it self (x – axis or y – axis or origin)

31.	The image of the point (2,5) by translation $(x, y) \rightarrow (x+2, y+1)$ is .... ((2, 5) or (4, 6) or (1, 4))
32.	The image of the point (1,3) by reflection in the x – axis is...... ((1, 3) or (1, −3) or (−1, 3))
33.	The image of the point (2,4) by reflection in the y – axis is .... ((2, 4) or (−2, 4) or (2, −4))
34.	The image of the point (−1, −4) by reflection in the .... is (1, –4) (y– axis or x – axis or origin point)
35.	The image of the point (3,2) by translation $(x, y) \rightarrow (x+3, y-2)$ is .... ((6, 0) or (3, 2) or (0, 0))
36.	The image of the point (– 2, −5) by translation $(x, y) \rightarrow (x-2, y)$ is .... ((2, 5) or (−4, −5) or (4, 5))
37.	The image of the point (3, −2) by translation $(x, y) \rightarrow (x, y+3)$ is .... ((3, 1) or (2, −2) or (3, 5))
38.	The image of the point (– 1, 2) by translation of magnitude of 3 units in the positive direction of the x-axis is .... ((−1, 5) or (2, 2) or (– 2, 2) or (– 1, 3))

39.	**The image of the point $(-3, 4)$ by translation of magnitude of 4 units in the negative direction of the y-axis is ....** **$((-3, 0)$ or $(-7, 4)$ or $(-3, 8)$ or $(-1, 4))$**
40.	**If À $(3,-3)$ is the image of A by translation $(x, y) \rightarrow (x-1, y-4)$, then the point A is ....** **$((2, -7)$ or $(4, 1)$ or $(-4, -1)$ or $(2, 1)$**
40.	**The image of the point $(-1, 4)$ by the translation $(3, -2)$ followed by reflection in the x-axis is ....** **$((2, 2)$ or $(-2, 2)$ or $(-2, -2)$ or $(2, -2)$**
41.	**If the point $(a, -1)$ is the image of $(2, 4)$ by the translation $(x, y) \rightarrow (x+1, y-b)$, then (a, b) is............** **$((3, 3)$ or $(1, 3)$ or $(3, 5)$ or $(1, -5))$**
42.	**The image of the point $(-3,-5)$ by reflection in the x -axis is ....** **$((3, -5)$ or $(3, 5)$ or $(-3, 5)$ or $(-3, -5)$**
43.	**The number of axes of symmetry of the equilateral triangle is ....** **(zero or 1 or 2 or 3)**
44.	**The image of the point $(2, -7)$ by reflection in the origin point is ....** **$((2, 7)$ or $(-2, 7)$ or $(-2, -7)$ or $(2, -7))$**

- **Polygons:**



Triangle (3 sides)	Quadrilateral (4 sides)	Pentagon (5 sides)	Heptagon (7 sides)	Octagon (8 sides)

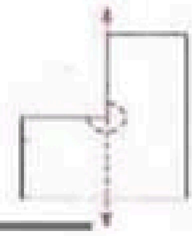
- **Convex polygon**

  Any interior angle of the convex polygon has measure less than 180°



- **Concave polygon :**

  There is at least one interior angle of concave polygon of measure more than 180° (reflex angle).



- The sum of measures of the **interior angles** of a polygon of n sides = (n−2)×180°
- The sum of measures of the **exterior angles** of a polygon of n sides = 360°

- **The regular polygon**

✓ A polygon is regular if all its **sides are equal** in length and all its **angles are equal** in measure.

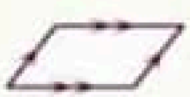
✓ The measure of **the interior angle** of a regular polygon → $x° = \frac{(n-2)\times 180}{n}$ — n is the number of sides

✓ **The number of sides** of a regular polygon → $n = \frac{360}{180-x}$ — x is the interior angle

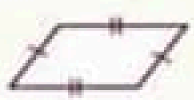
✓ The number of diagonals polygon= $\frac{n(n-3)}{2}$

A quadrilateral is a parallelogram if one of the following cases is satisfied

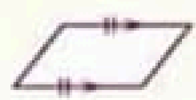
- Each two opposite sides are parallel



- Each two opposite sides are equal in length



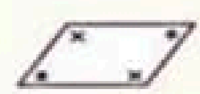
- Two opposite side are parallel and equal in length



- The two diagonals bisect each other



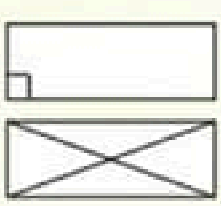
- Each two opposite angles are equal in measure



A parallelogram is a

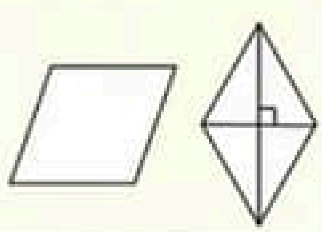
Rectangle

If :
One of its angles is right.
or
The diagonals are equal in length.



Rhombus

If :
Two adjacent sides are equal in length.
or
Its diagonals are perpendicular.



Square

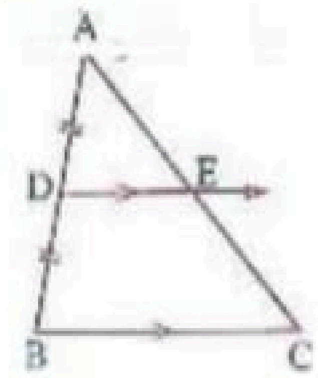
If :
One of its angles is right and two adjacent sides are equal in length.
or
One of its angles is right and its diagonals are perpendicular.
or
The diagonals are equal in length and perpendicular.
or
Two adjacent sides are equal in length and its diagonals are egual in length.

Notice that :

- A square is a rectangle with two adjacent sides equal in length.
- A square is a rhombus with a right angle.
- To prove that the quadrilateral is a rectangle , a rhombus or a square , we must first prove that it is a parallelogram.

➢ **Theorem (2):**

The ray drawn from the midpoint of a side of a triangle parallel to another side **bisects the third side**.

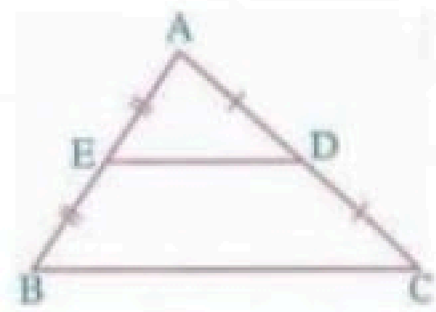


$\because$ D is the midpoint of $\overline{AB}$, $\because$ $\overline{DE} \parallel \overline{CB}$

$\therefore$ E is the midpoint of $\overline{AC}$

➢ **Corollary**

The line segment joining the midpoints of two sides of a triangle is **parallel to the third side**

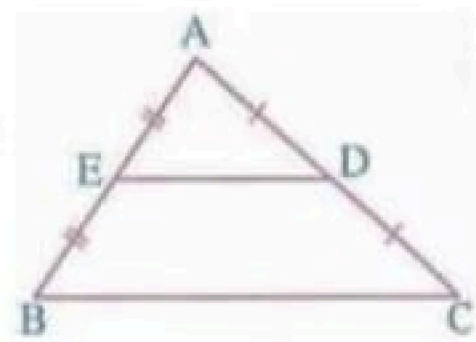


$\because$ D is the midpoint of $\overline{AC}$, $\because$E is the midpoint of $\overline{AB}$

$\therefore$ $\overline{DE} \parallel \overline{CB}$

➢ **Theorem (3):**

The length of the line segment joining the midpoints of two sides of a triangle is **equal to half the length of the third side**.



$\because$ D is the midpoint of $\overline{AC}$, $\because$E is the midpoint of $\overline{AB}$

$\therefore$ DE =½ BC

➢ **Reflection in a coordinate plane**

✓ The image of a point (x , y) by reflection in **the x–axis** is (x, –y)

✓ The image of a point (x , y) by reflection in **the y–axis** is (–x, y)

✓ The image of a point (x , y) by reflection in **the origin point** is (–x, –y)

➢ **Ex: Draw on a square lattice Δ ABC where A (9 , 5) , B (6 , 1) and C (2 , 2)**

✓ **Draw Δ A\`B\`C\` which is the image of Δ ABC by reflection in x–axis**

✓ **Draw Δ A\`\`B\`\`C\`\`which is the image of Δ ABC by reflection in y–axis**

Reflection in x–axis

A (9 , 5) ⟶ A\` (9 , –5)

B (6 , 1) ⟶ B\` (6 ,– 1)

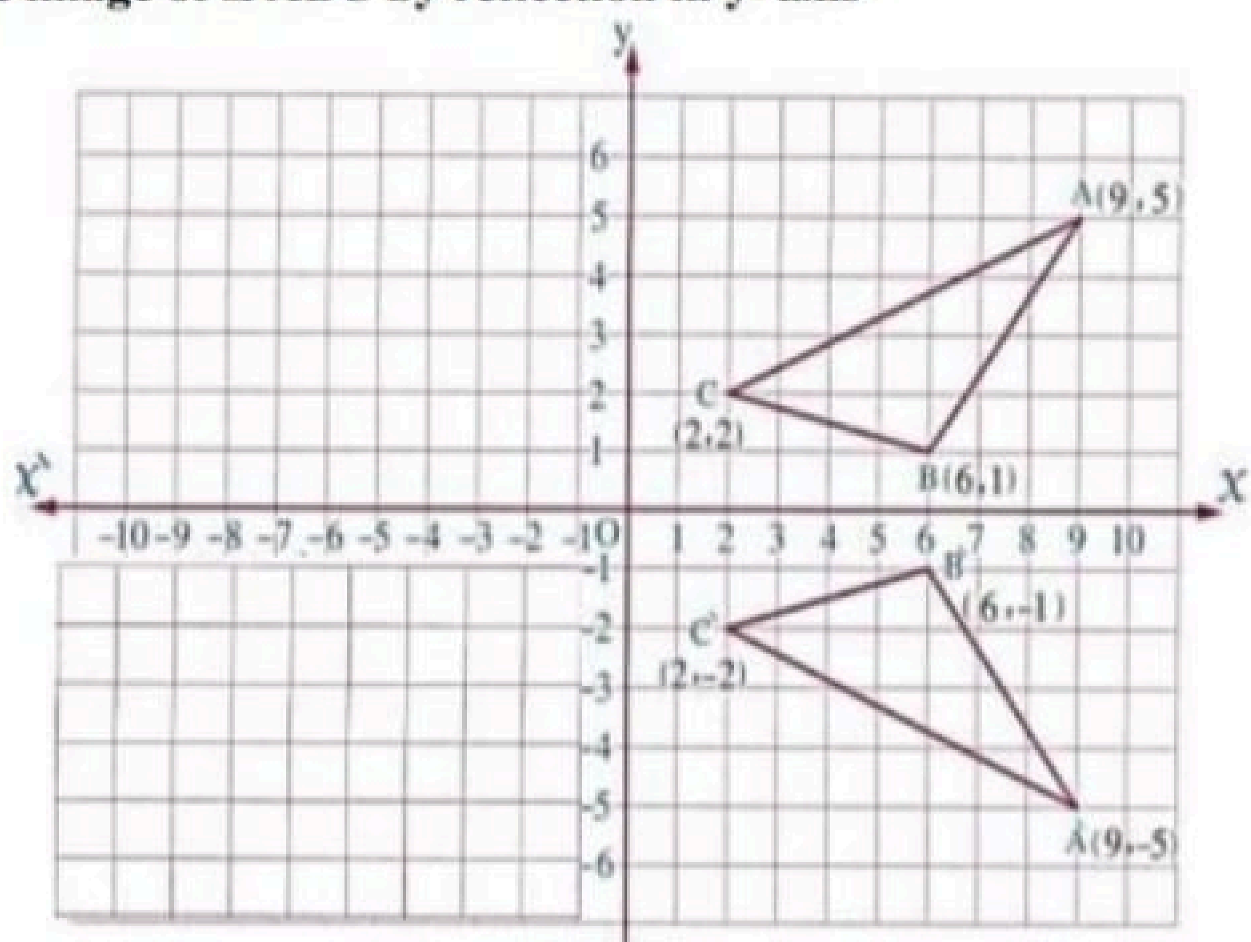
C (2 , 2) ⟶ C\` (2 , –2)

Reflection in y–axis

A (9 , 5) ⟶ A\`\` (–9 , 5)

B (6 , 1) ⟶ B\`\` (–6 , 1)

C (2 , 2) ⟶ C\`\` (–2 , 2)



Draw Δ ABC where A (4 , 1) , B (2 , 4) and C (– 1 , 3) , then map its image by reflection in the origin point.

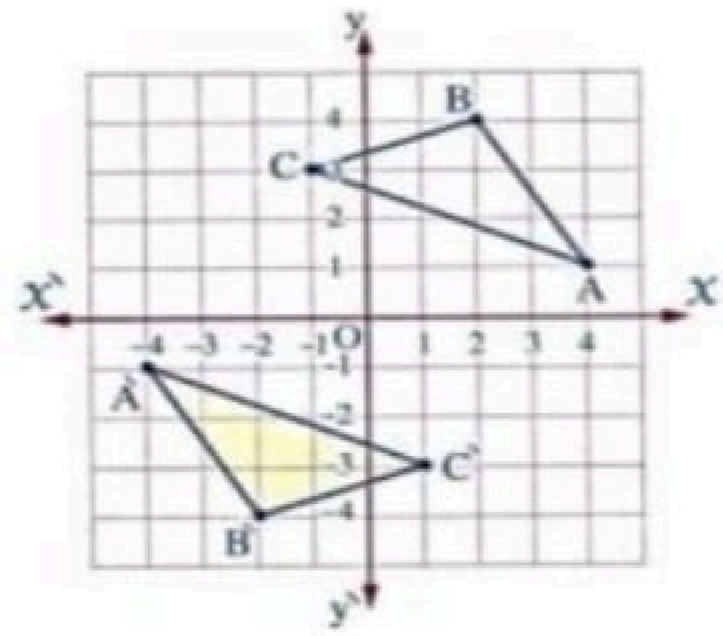
Answer

$\because (x , y) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (-x , -y)$

$\therefore A (4 , 1) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \grave{A} (-4 , -1)$

$, B (2 , 4) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \grave{B} (-2 , -4)$

$, C (-1 , 3) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \grave{C} (1 , -3)$



- **Translation in coordinate plane :**

✓ The image of the point A by translation $A (x , y) \xrightarrow{\text{by translation } (a , b)} A' (x + a , y + b)$

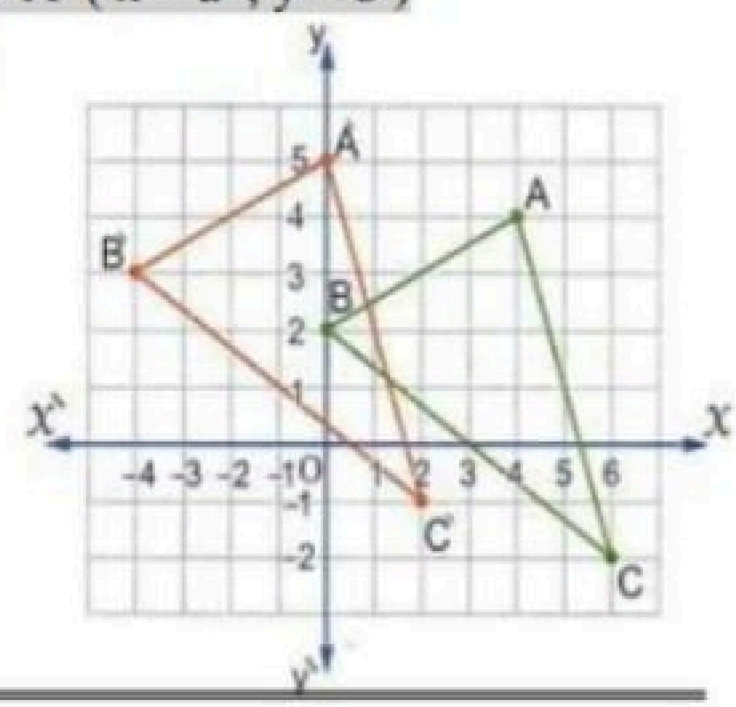
Ex_2 : Draw on a square lattice Δ ABC where A (4 , 4) , B (0 , 2) and C (6 , – 2) then , find its image by translation (x , y) → (x – 4 , y + 1)

Answer

$A (4 , 4) \xrightarrow{(x-4 , y+1)} A' (0 , 5)$

$B (0 , 2) \xrightarrow{(x-4 , y+1)} B' (-4 , 3)$

$C (6 , -2) \xrightarrow{(x-4 , y+1)} C' (-2 , -1)$



- **Rotation in coordinate plane :**

The point		
(x , y)	by rotation R (O , 90°) ($\frac{1}{4}$ turn)	→ The point (– y , x)
	by rotation about O with an angle of measure (– 90°) or (270°)	→ The point (y , – x)
	by rotation R (O , ± 180°) ($\frac{1}{2}$ turn)	→ The point (– x , – y)
	by rotation R (O , ± 360°) (identity rotation)	→ The point (x , y)

On a square lattice , draw Δ ABC where A (1 , – 1) , B (2 , 2) and C (4 , 3) :

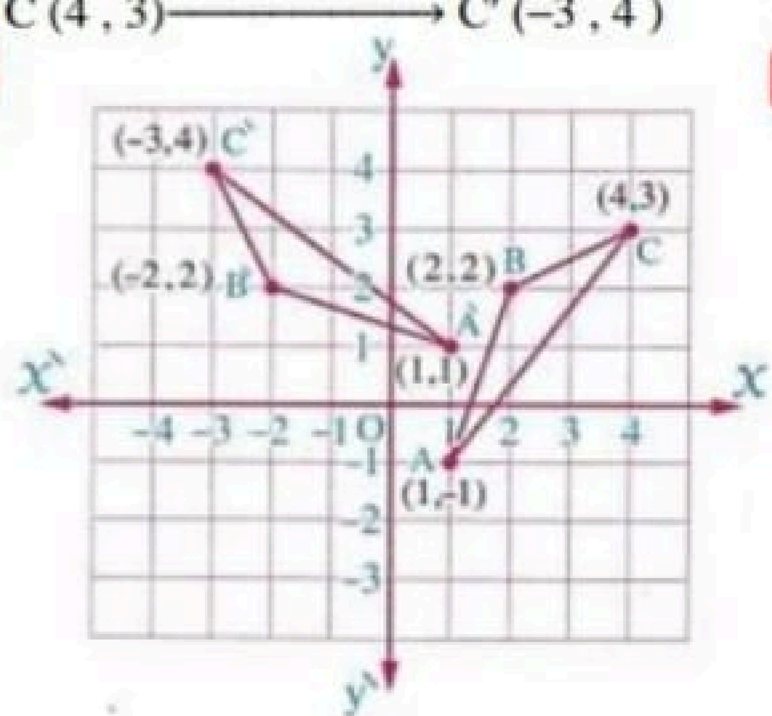
1. Draw $\Delta \grave{A}\grave{B}\grave{C}$ which is the image of Δ ABC by rotation R (O , 90°)
2. Draw $\Delta \ddot{A}\ddot{B}\ddot{C}$ which is the image of Δ ABC by rotation R (O , 180°)

$A (1 , -1) \xrightarrow{R\ (0 , 90)} A' (1 , 1)$ $\quad$ $A (1 , -1) \xrightarrow{R\ (0 , 180)} A'' (-1 , 1)$

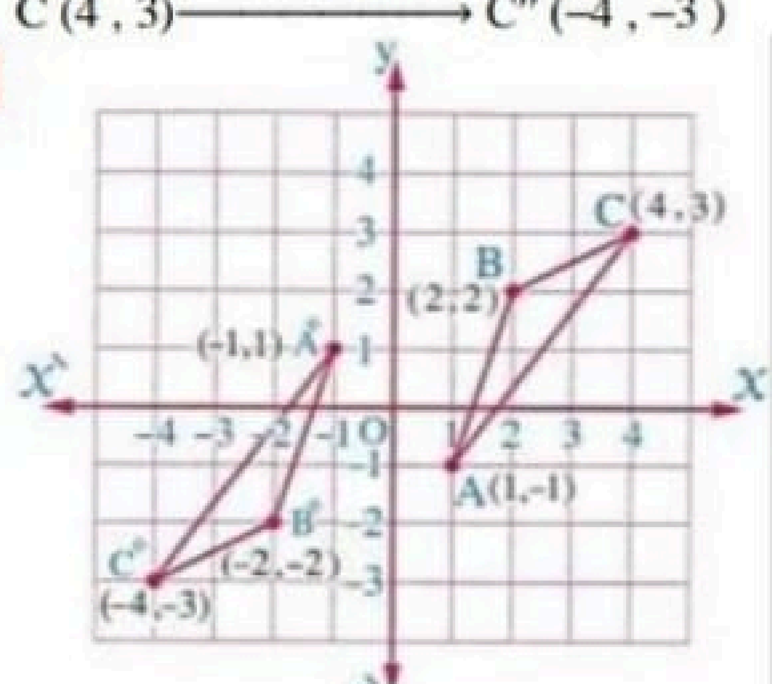
$B (2 , 2) \xrightarrow{R\ (0 , 90)} B' (-2 , 2)$ $\quad$ $B (2 , 2) \xrightarrow{R\ (0 , 180)} B'' (-2 , -2)$

$C (4 , 3) \xrightarrow{R\ (0 , 90)} C' (-3 , 4)$ $\quad$ $C (4 , 3) \xrightarrow{R\ (0 , 180)} C'' (-4 , -3)$

1



2



1) Choose

a) The sum of the measures of the interior angles of a triangle equals --------- ($90°$, $180°$, $270°$, $360°$)

b) The image of the point (- 1 , 3) by translation (4 , - 2) is ---------- [(3 , -1) , (5 , 1), (5 , -5) , (3 , 1)]

c) The measure of the exterior angle of the equilateral triangle is ---------- ($30°$, $45°$, $60°$, $120°$)

d) In a parallelogram if the adjacent sides are equal in the length , then the shape is --------

(square , rhombus , rectangle , trapezium)

e) The number of the diagonals of a pentagon is -------- (3 , 5 , 7 , 9)

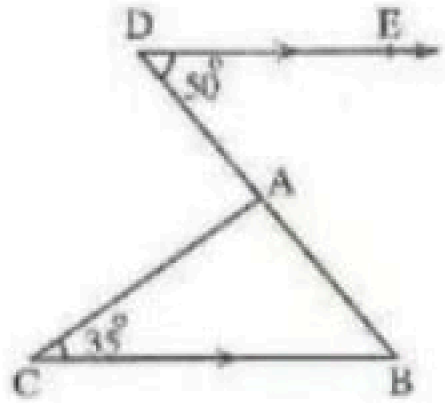
In the opposite figure :

$\overrightarrow{DE} \parallel \overline{CB}$, m (∠ D) = 50°

, m (∠ C) = 35°

Find : (1) m (∠ B)

(2) m (∠ BAC)



Proof

∵ $\overrightarrow{DE} \parallel \overline{CB}$

∴ ∠B = ∠ D = 50° (alternate)

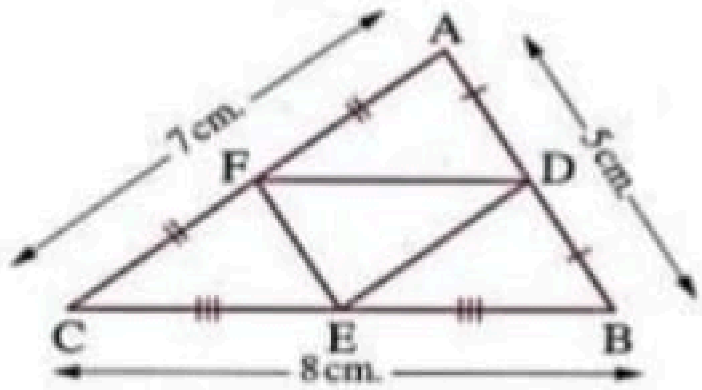
∴ ∠BAC = 180 – (35 + 50) = 95°

In the opposite figure :

AB = 5 cm., BC = 8 cm.,

AC = 7 cm., D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.

Calculate the perimeter of : Δ DEF



Proof∵ D is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{AC}$

∴ FD // BC ∴ FD =½ BC ∴ FD =½ ×8 = 4 cm

∵ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BC}$

∴ DE // AC ∴ DE =½ AC ∴ DE =½ ×7 = 3.5 cm

∵F is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

∴FE // AB ∴FE =½ AB∴ FE =½ ×5 = 2.5 cm

∴ The perimeter = 4 + 3.5 + 2.5 = 10 cm

---

2) Complete

a) The image of the point (2 , 1) by reflection in x-axis is --------------

b) The image of the point (2 , - 1) by rotation about the origin point with an angle ofmeasure $180°$ is -------

c) The square is a rectangle in which ---------

d) ABCD is a parallelogram in which m (∠ A) = $60°$, then m (∠ B) = ----------

e) The image of the point (5 ,3) by translation : (x , y) ⟶ (x + 3 , y - 1) is -----------

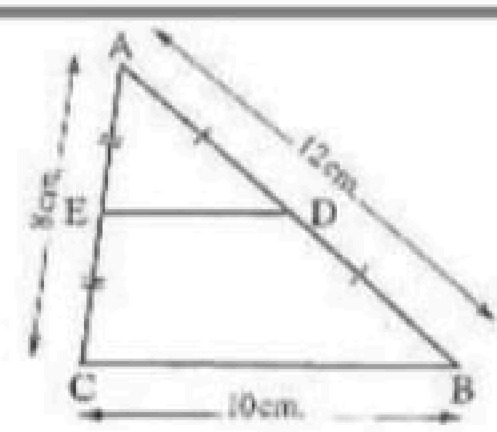
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In the opposite figure :

Δ ABC in which : AB = 12 cm. ,

BC = 10 cm. , AC = 8 cm.

Find the perimeter of : Δ ADE



Proof∵ D is the midpoint of $\overline{AB}$,

E is the midpoint of $\overline{AC}$

∴ED // BC , ED =½ BC

∴ED =½ ×10 = 5 cm

∴ Per of Δ ADE = 4 + 5 + 6=15

---

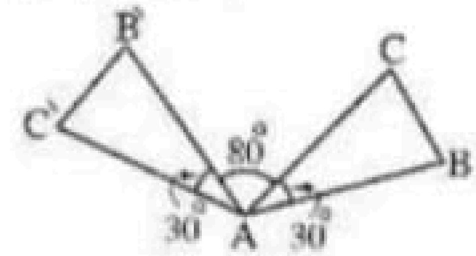
3) Choose :

a) The image of the point (3, – 5) by reflection in y-axis is ---------- [(3 , 5) , (–3 , 5) , (–3 , 5) , (–5 , 3)]

b) The sum of the measures of interior angles of a pentagon is -------- ($360°$, $450°$, $540°$, $720°$)

c) The number of diagonals of quadrilateral is -------- (2 , 3 , 4 , 5)

d) In the opposite figure : Δ AB`C` is the image of Δ ABC by rotation about A with an angle of measure ---------- ($30°$, $80°$, $110°$, $140°$)



e) The diagonal of a square divided its vertex angle in two angles the measure of each ofthem is -------- ($30°$, $45°$, $60°$, $90°$)

f) The image of the point (2 , - 5) by reflection in x-axis is ---------- [(2 , - 5) , (2 , 5) , (2 ,- 5) , (5 , 2)]

g) The measure of each angle of regular hexagon equals --------- ($60°$, $108°$, $120°$, $135°$)

In the opposite figure :

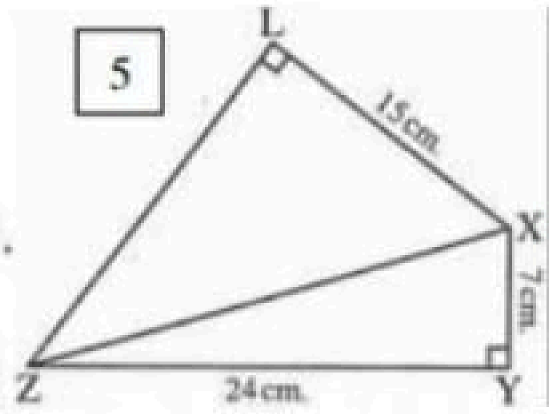
XYZL is a quadrilateral in which

m (∠ Y) = m (∠ L) = 90° , XY = 7 cm.

YZ = 24 cm. , XL = 15 cm.

Find the length of each of : $\overline{XZ}$, $\overline{LZ}$

5



Proof in ΔXYZ ,∠ Y = 90

∴ $(XZ)^2 = (XY)^2 + (YZ)^2$

$= (7)^2 + (24)^2 = 625$ ∴ XZ = 25

In Δ LXZ ,∠ L = 90

∴ $(LZ)^2 = (XZ)^2 - (LX)^2$

$= (25)^2 - (15)^2 = 400$ ∴ LZ = 20

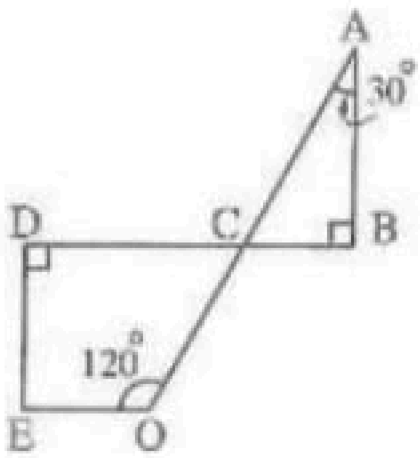
In the opposite figure :

$\overline{AB}$ and $\overline{ED}$ are perpendicular to $\overline{BD}$, $\overline{BD} \cap \overline{AO} = \{C\}$,

m (∠ A) = 30°

, m (∠ EOC) = 120° ,

Find : m (∠ E)



Proof in ΔABC ,

∵ ∠ ACB = 180 – (90 + 30) = 60

∴ ∠ ACB = ∠ DCO = 60 (V.O.A)

∵Sum of interior angles of DCOE = 360

∴ ∠ E = 360 – (90 + 120 + 60) = 90

---

4) **Complete :**

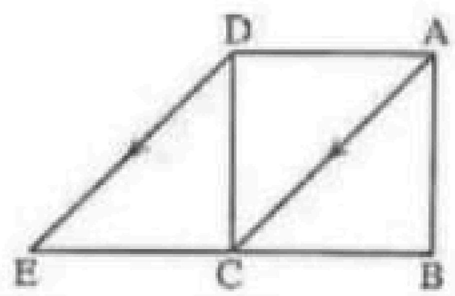
a) The sum of the measures of the angles of the quadrilateral equals --------------
b) The image of the point (2 ,3) by translation MN in direction $\overrightarrow{MN}$ Where M (2 , - 1) and N (5 , 1) is --------
c) ABCD is a parallelogram in which m (∠ A) = 60° , then m (∠ B) = ----------
d) The two diagonals are equal in the length and not perpendicular in --------
e) The ray drawn parallel to one side of a triangle and passing through the midpoint of another side ---------

---

In the opposite figure :

ABCD is a square , E ∈ $\overrightarrow{BC}$, $\overline{AC} \parallel \overline{DE}$

Prove that : ACED is a parallelogram.



Proof∵ ABCD is a square

∴ $\overline{DA} \parallel \overline{CB}$, ∵ E ∈ $\overline{BC}$

∴ $\overline{DA} \parallel \overline{EC}$, ∵ $\overline{AC} \parallel \overline{DE}$

∴ ACED is a parallelogram

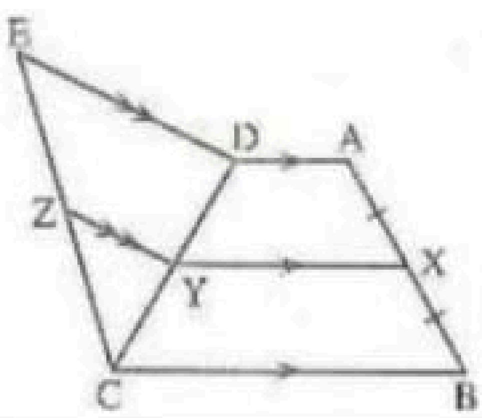
In the opposite figure :

X is the midpoint of $\overline{AB}$

, Y ∈ $\overline{CD}$, Z ∈ $\overline{CE}$

, $\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$, $\overline{YZ} \parallel \overline{DE}$

is CZ = ZE ? giving reason



Proof in the figure ABCD

∵ X is the midpoint of $\overline{AB}$

∵ $\overline{DA} \parallel \overline{YX} \parallel \overline{CB}$

∴ Y is the midpoint of $\overline{DC}$

In ΔEDC ,∵Y is the midpoint of $\overline{DC}$, ∵ $\overline{ED} \parallel \overline{ZX}$

∴ Z is the midpoint of $\overline{EC}$ ∴ CZ =ZE

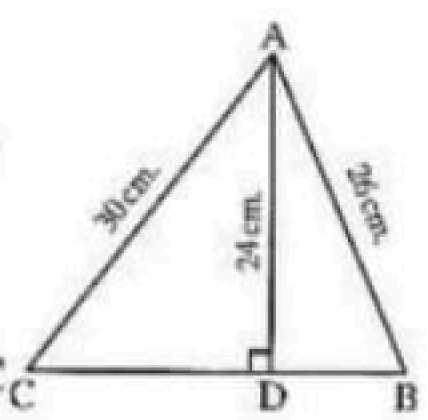
In the opposite figure :

$\overline{AD} \perp \overline{BC}$, if AD = 24 cm. ,

AB = 26 cm. , AC = 30 cm.

(1) **Find the length of :** $\overline{BC}$

(2) **Find the area of :** Δ ABC



Proofin Δ ADB ,∠ ADB = 90

∴ $(DB)^2 = (AB)^2 - (AD)^2 = (26)^2 - (24)^2 = 100$

∴DB = 10

In Δ ADC ,∠ ADC = 90

∴ $(CD)^2 = (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324$

∴CD = 18

∴ BC = 10 + 18 = 28 , Area= ½ × 28 × 24 = 336 cm^2

---

5) **Complete :**

a) The rhombus is a parallelogram in which ---------------------------
b) Each opposite angles in a parallelogram are ----------------------
c) (– 3, 2) is the image of the point (3 , 2) by reflection in ----------- axis.
d) The line segment joining the midpoints of two sides of a triangle is --
e) The image of the point (4 , 6) by geometric transformation (x, y) ⟶ (–x , y – 7) is -----------
f) The image of the point (3 , 2) by rotation with an angle of measure 180° about the origin is ------------
g) The quadrilateral is a parallelogram if --------------------------

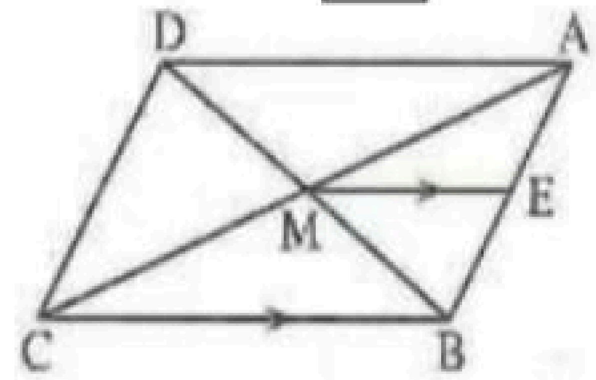
6

In the opposite figure :

ABCD is a parallelogram , M is the intersection point of its diagonals

Draw $\overrightarrow{ME} \parallel \overline{CB}$

is AE = EB ? giving reason.



Proof in the figure ABCD

∵ABCD is a parallelogram; M is the intersection of its diagonals.

In ABC ∵ M is the midpoint of $\overline{AC}$

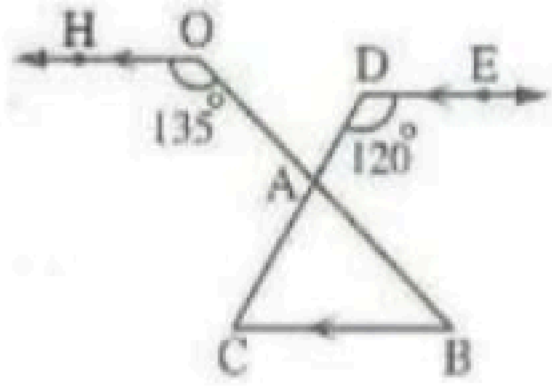
∵ $\overline{ME} \parallel \overline{CB}$

∴ E is the midpoint of $\overline{AB}$, ∴ AE = EB

In the opposite figure :

$\overrightarrow{DE} \parallel \overrightarrow{OH} \parallel \overline{BC}$, m (∠ ADE) = 120°

, m (∠ AOH) = 135° , Calculate the measures of the angles of the triangle ABC



Proof ∵ $\overline{DE} \parallel \overline{CB}$

∴ ∠C = 180 − 120 = 60

∵ $\overline{HO} \parallel \overline{CB}$

∴ ∠B = 180 − 135 = 45

∴ ∠CAB = 180 − (60 + 45) = 75

---

6) **Choose :**

a) The parallelogram Whose diagonals are perpendicular and not equal in length is called -----------
(rhombus , square , rectangle , trapezium)

b) The measure of each angle of a regular pentagon is --------- (90° , 108° , 120° , 136°)

c) The triangle contains at least two -------------- angles (acute , obtuse , right , reflex)

d) If ABCD is a parallelogram in which BC = 8 cm , CD = 6 cm ,then its perimeter = -----------
(14 cm , 28 cm , 48 cm , 56 cm)

e) The image of the point (2 , - 1) by reflection in x-axis is --------[(2 , 1) , (1 , 2) , (-2 , -1) , (-1, 2)]

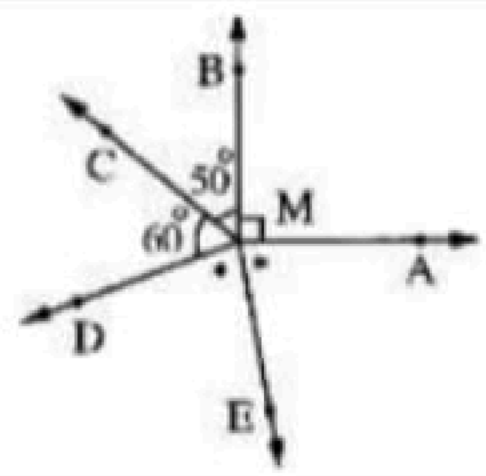
In the opposite figure :

$\overrightarrow{MB} \perp \overrightarrow{MA}$, m (∠ BMC) = 50°

m (∠ CMD) = 60°

, if $\overrightarrow{ME}$ bisects ∠ AMD

Find : m (∠ AME)



Proof

∵ Sum of angles at a point = 360

∴ ∠ AMD = 360 − (90 + 50 + 60) = 160

∵$\overrightarrow{ME}$ bisects ∠ DMA

∴ ∠ AME = 160 ÷ 2 = 80

In the opposite figure :

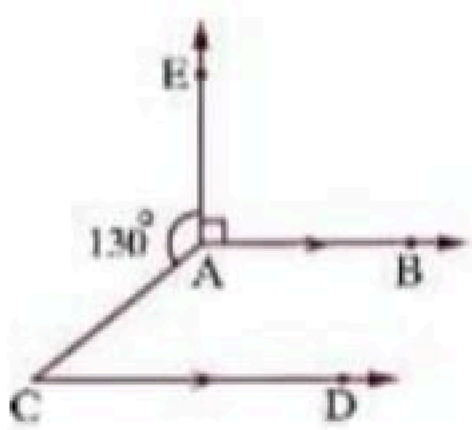
$\overrightarrow{AB} \parallel \overrightarrow{CD}$,

, m (∠ EAC) = 130° and

m (∠ EAB) = 90°

Find : (1) m (∠ BAC)

(2) m (∠ C)



Proof

∵ Sum of angles at a point = 360

∴ ∠ BAC = 360 − (90 + 130) = 140

∵$\overline{AB} \parallel \overline{CD}$

∴ ∠ C = 180 − 140 =40 (interior)

---

7) **Complete :**

a) The line segment joining the midpoints of two sides of a triangle is ---------

b) In a parallelogram each opposite angles are -------------

c) The image of the point (5 , - 3) by translation 3 units in negative direction of x-axis is ------------

---

In the opposite figure :

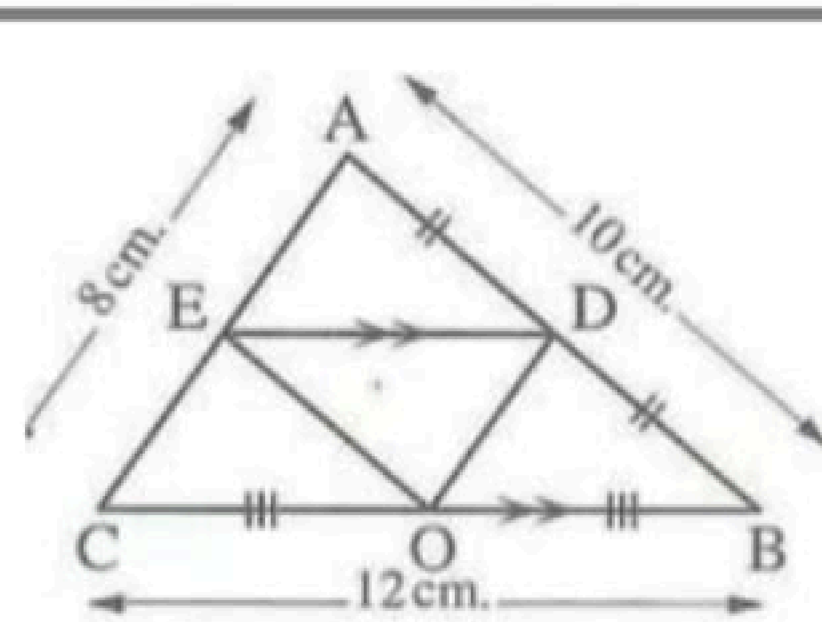
ABC is a triangle in which D is the midpoint of $\overline{AB}$, O is the midpoint of $\overline{BC}$, E ∈ $\overline{AC}$

Such that $\overline{DE} \parallel \overline{BC}$,

AB = 10 cm. , BC = 12 cm. , AC = 8 cm.

(1) **Prove that :** DBOE is a parallelogram.

(2) **Find the perimeter of :** Δ EDO



Proof

∵ D is the midpoint of $\overline{AB}$,∵ED // BC

∴E is the midpoint of $\overline{AC}$

∴ED =½ BC ∴ED = OB (2)

∵ED // BC , ED = OB ∴ DBOE is a ▱

∴ED =½ ×12 = 6 cm

∵ D is the midpoint of $\overline{AB}$, O is the midpoint of $\overline{BC}$

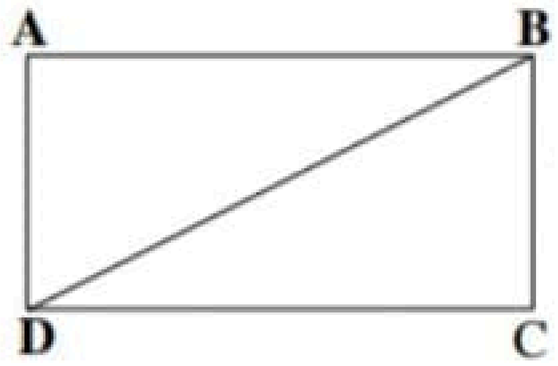
∴ DO// AC , DO =½ AC ∴ DO =½ ×8 = 4 cm

∵E is the midpoint of $\overline{AC}$, O is the midpoint of $\overline{BC}$

∴EO // AB , EO =½ AB ∴ EO =½ ×10 = 5 cm

∴ The perimeter of Δ EOD = 4 + 6 + 5 = 15 cm

8) **Find the length of the diagonal of a rectangle whose area is 48 cm^2 and of width 6 cm.**



L = area ÷ w = 48 ÷ 6 = 8 cm

$\therefore (DB)^2 = (BC)^2 + (DC)^2$

$= (6)^2 + (8)^2 = 100 \quad \therefore DB = 10$

---

9) **If the point A is the image of point B (- 1 , 2) by reflection in y-axis s find the image of A by translation (- 1 , 2)**

Then the point A is (1 , 2)

Then A + T = A`

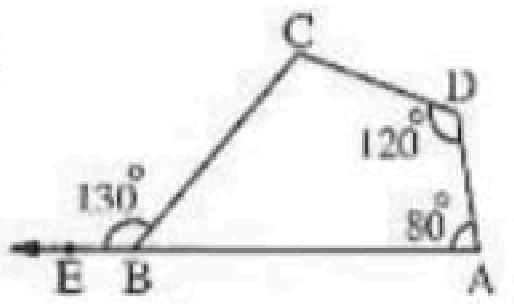
= (1 , 2) + (- 1 , 2) = (0,4)

In the opposite figure :

m (∠ A) = 80° , m (∠ D) = 120°

, m (∠ CBE) = 130°

Find : m (∠ C)



Proof∵ ∠CBA = 180 – 130 = 50

∵ Sum of interior angles of ABCD = 360

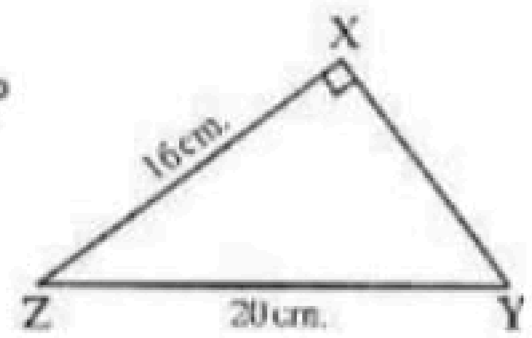
∴ ∠ C =360 – (80 + 120 +50) = 110

In the opposite figure :

XYZ is a triangle in which m (∠ X) = 90°

YZ = 20 cm. , XZ = 16 cm.

Find the length of : $\overline{XY}$



Proof

In Δ XYZ ,∠ X = 90

$\therefore (XY)^2 = (ZY)^2 - (ZX)^2$

$= (20)^2 - (16)^2 = 144$

∴XY = 12

---

10) **Choose :**

a) The image of the point (- 3 , 4) by reflection in y-axis is ---------[(3 , -4) , (3 , 4) , (-3 , -4) , (4 , -3)]

b) The sum of the measures of the exterior angles of a triangle equals -------(90° , 108° , 180° , 360°)

c) The diagonals are equal in the length and perpendicular in ------------
(square , rhombus , rectangle , parallelogram)

d) The image of the point (- 3 , 5) by rotation about the origin and with an angle of measure 90° is --------
[(5 , 3) , (- 5, 3) , (3 , 5) , (- 5 , - 3)]

e) The measure of each angle of regular octagon equals ----------- (108° , 120° , 135° , 144°)

f) The measure of the exterior angle of a triangle is --

g) The parallelogram whose diagonals are equal in length and perpendicular is ------------

h) ABCD is a parallelogram in which m (∠ A) = 50° , then m (∠ B) = -------------

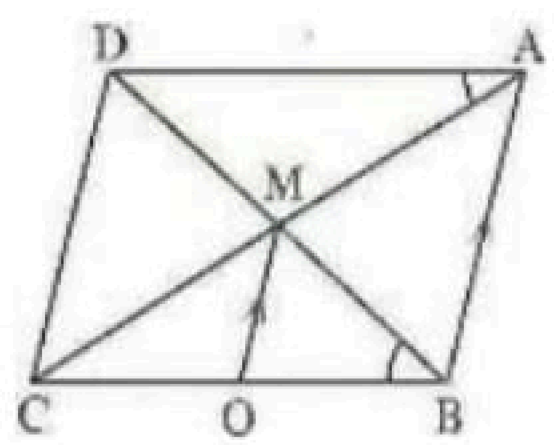
---

In the opposite figure :

ABCD is a parallelogram its diagonals intersect at M , $\overrightarrow{MO} // \overline{AB}$

, $\overline{MO} \cap \overline{BC} = \{O\}$

Prove that : BO = OC



Proof in the figure ABCD

∵ABCD is a parallelogram, M is the intersection of its diagonals.

In ABC ∴ M is the midpoint of $\overline{AC}$

∵ $\overline{MO} // \overline{AB}$

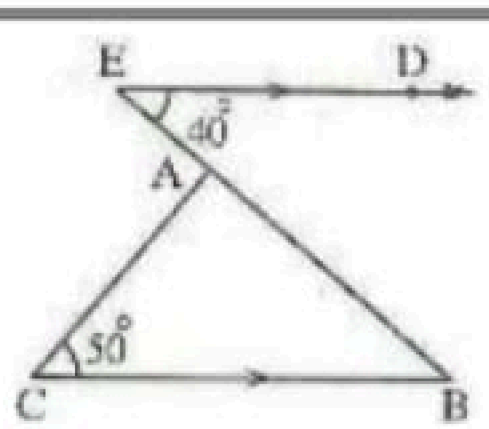
∴ O is the midpoint of $\overline{BC}$,∴BO = OC

In the opposite figure :

$\overrightarrow{ED} // \overrightarrow{CB}$, m (∠ C) = 50°

, m (∠ E) = 40°

Prove that : $\overline{AC} \perp \overline{BE}$



Proof∵ $\overrightarrow{DE} // \overrightarrow{CB}$

∴ ∠B =∠ E =40° (alternate)

∴ ∠BAC = 180 – (50 + 40) = 90

∴ $\overline{AC} \perp \overline{BE}$

8

11) Complete :

a) The parallelogram whose diagonals are perpendicular is --------------

b) If the measure of an interior angle of a triangle is equal to the sum of the measures ofthe other two interior angles , then the triangle is ------------------

c) Any triangle has at least two ------------- interior angles.

d) The image of the point (2 , - 4) by reflection in x-axis is ------------

e) The image of the point (3 , - 2) by translation $(x, y) \longrightarrow (x - 1 , y + 6)$ is ---------

f) The image of the point (- 4 , 5) by translation (2 , - 3) is ----------------

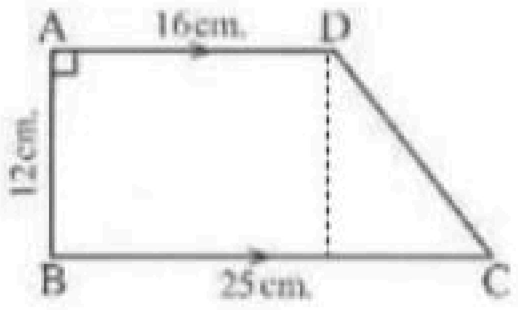
g) ΔXYZ , $m(\angle Y) = 90°$, then $(XZ)^2 =$ -----------

In the opposite figure :

ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$

, $m(\angle A) = 90°$, AB = 12 cm. , BC = 25 cm. , AD = 16 cm.

Find the length of : $\overline{DC}$



Ans :

Draw $\overline{DX} \perp \overline{BC}$

DX = 12

XC = 25 − 16 = 9

$\therefore (DC)^2 = (BX)^2 + (XC)^2$

$= (12)^2 + (9)^2 = 225$

$\therefore DC = 15$

---

12) Choose

a) The measure of each angle of the regular pentagon is -------- (90° , 108° , 120° , 144°)

b) The smallest number of the acute angle in any triangle is ----------- (zero , 1 , 2 , 3)

c) The rhombus of diagonals are equal in length is ----------------
(square , rectangle , parallelogram , trapezium)

d) The image of the point (2 , - 1) by reflection in x-axis is ---------[(2 , 1) , (1 ,2) , (-2 ,- 1) , (-1 ,2)]

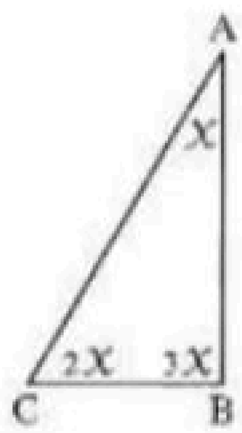
e) The image of the square by rotation about the origin point with an angle of measure90° is ---------
(rectangle , square , rhombus , trapezium)

f) The image of a triangle by rotation about the origin point with an angle of measure180° is --------
(triangle , line segment , point , straight line)

g) The image of the point (3 , - 2) by reflection in y-axis ------ [(3 ,2) , (-3 ,-2) , (-3 , 2) , (-2, 3)]

In the opposite figure :

Find with proof the measures of the angles of ΔABC



Proof$\because$ Sum of angles of $\Delta = 180$

$\therefore x + 2x + 3x = 180$ $\therefore \angle A = 30$

$\therefore 6x = 180$ $\therefore \angle B = 3x = 90$

$\therefore x = 180 \div 6 = 30 \therefore \angle B = 2x = 60$

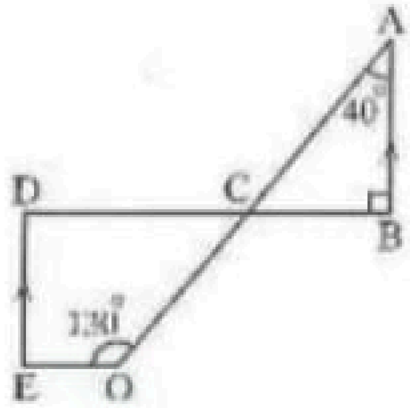
In the opposite figure :

$\overline{BD} \cap \overline{AO} = \{C\}$, $\overline{AB} \parallel \overline{DE}$,

$m(\angle A) = 40°$, $m(\angle B) = 90°$

$m(\angle COE) = 130°$

Find : $m(\angle E)$



Proof$\because$ $\angle ACB = 180 - (90+40) = 50$

$\because \angle ACB = \angle DCO = 50$ (V.O.A)

$\because \overline{AB} \parallel \overline{CB}$ $\therefore \angle B = \angle D = 90$ (alternate)

$\because$Sum of interior angles of CDEO = 360

$\therefore \angle E = 360 - (130+90+50) = 90$

13) Choose

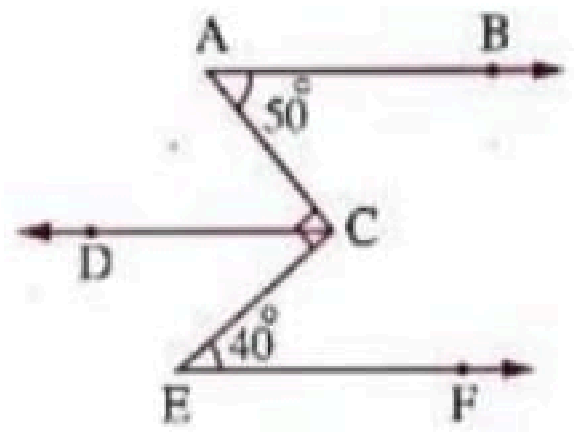
a) The measure of each angle of the regular hexagon is -------- (108° ,120° , 136°,144°)
b) Any triangle has at least two ---------- interior angles. (acute , right , obtuse , straight)
c) The diagonals of the rectangle are ----------
(parallel , perpendicular , equal in the length , equal in length & perpendicular)

14) Complete :

a) The sum of the measures of the interior angles of a pentagon equals --------------------
b) The measure of the exterior angle of an equilateral triangle at one of itsvertices -------------
c) The rectangle is a parallelogram in which one of its angles is ----------
d) Theimage of the point (- 4 , 5) by translation (2 , - 3) is ---------------
e) If the image of the point (4 , 0) by rotation about the origin point is (0 , - 4) , the measure of rotation angle is -----------

In the opposite figure :

$\overrightarrow{AB} // \overrightarrow{CD}$, m (∠ A) = 50° ,
∠ ACE is right and m (∠ E) = 40°
Prove that : $\overrightarrow{AB} // \overrightarrow{EF}$



Proof: ∵ $\overline{AB} // \overline{CD}$
∴∠A =∠ ACD = 50 (alternate)
∴∠ DCE = 90 – 50 = 40
∵∠ DCE =∠E = 40 (alternate)
∴ $\overline{DC} // \overline{EF}$
∵ $\overline{AB} // \overline{CD}$ ∴ $\overline{AB} // \overline{EF}$

15) Choose:

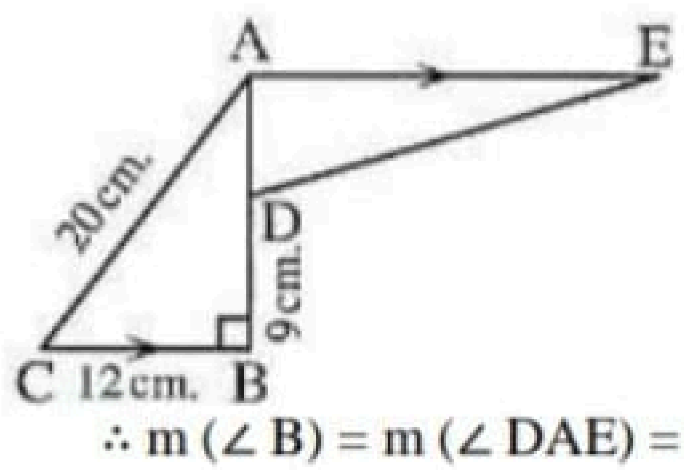
a) The diagonal of square makes an angle of measure ---------- with any of its sides (45° , 60° , 90° , 120°)
b) The measure of any of the exterior angle of an equilateral triangle equals ------- (60° , 90° , 120° , 180°)
c) In Δ ABC if m (∠ A) = m (∠ B) + m (∠ C) then the angle A is --------- (acute , right , obtuse , straight)
d) The sum ofthe measures of any two consecutive angles in a parallelogram equals ---------
(90°,180° , 270° , 360°)
e) The image of the point (5 , - 3) by rotation about the origin point is itself , then themeasure of rotation angle is ---------- (90°, 180° , 270° , 360°)

16) Complete:

a) The length of a line segment joining the midpoints of two sides of a triangle equals ------------
b) The parallelogram Whose diagonals are perpendicular and equal in length is ---------------
c) The parallelogram whose perimeter is 24 cm. and the length of one of its sides is 7 cm , then the length of its adjacent side equals --------------------
d) The image of the point (3 , 4) by reflection in the x-axis is --------------- and its image byreflection in the y-axis is -----------------
e) If the image of the point (- 1 , 3) by a translation is (1 , 4) , then the image of the point(3 , - 2) by the same translation is -----------------

In the opposite figure :

Δ ABC in which m (∠ B) = 90° , $\overline{AE} // \overline{BC}$,
if BC = 12 cm. , AC = 20 cm. , D ∈ $\overline{AB}$
such that : BD = 9 cm. , AE = 2 BC ,
Find the length of each of : $\overline{AD}$, $\overline{ED}$



Proof :

In Δ ABC is right-angled at B

∴ $(AB)^2 = (AC)^2 - (BC)^2 = (20)^2 - (12)^2 = 256$

∴ AB = 16 , then AD = 16 – 9 = 7 cm

∵ $\overline{AE} // \overline{BC}$

∴ m (∠ B) = m (∠ DAE) = 90

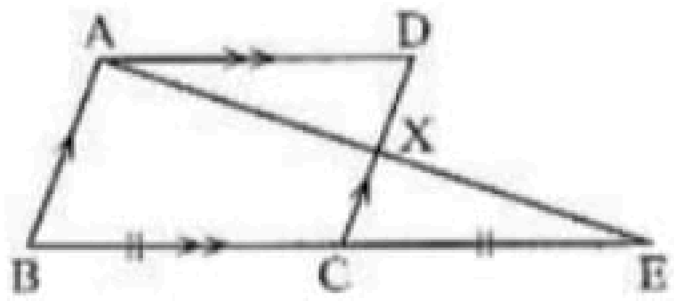
In Δ ADE is right-angled at A

AE = 2 BC = 24 cm

∴ $(DE)^2 = (AE)^2 + (AD)^2 = (24)^2 + (7)^2 = 625$

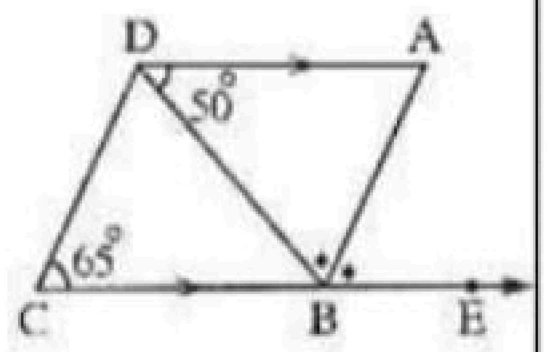
∴DE = 25 cm

ABCD is a parallelogram, $E \in \overrightarrow{BC}$, $CE = BC$, $\overline{AE} \cap \overline{DC} = \{X\}$
Prove that AX = XE



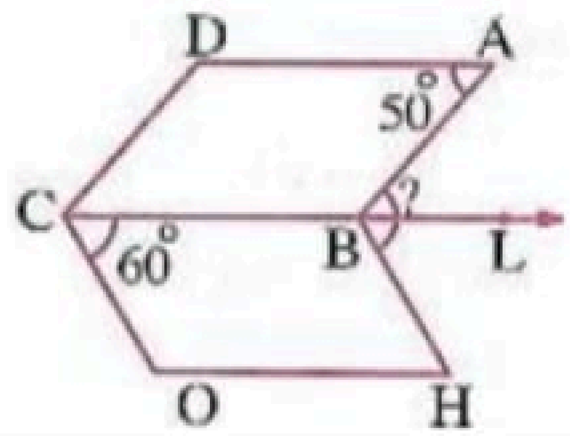
Proof ∵ ABCD is a parallelogram
∵ C is midpoint of $\overline{BE}$
∵ $\overline{XC} \parallel \overline{AB}$
∴ C is midpoint of $\overline{AE}$
∴ AX = XE

In the opposite figure :
$\overrightarrow{DA} \parallel \overleftrightarrow{BE}$, $\overrightarrow{BA}$ bisects ∠ DBE, m (∠ ADB) = 50°, m (∠ C) = 65°
Prove that : ABCD is a parallelogram



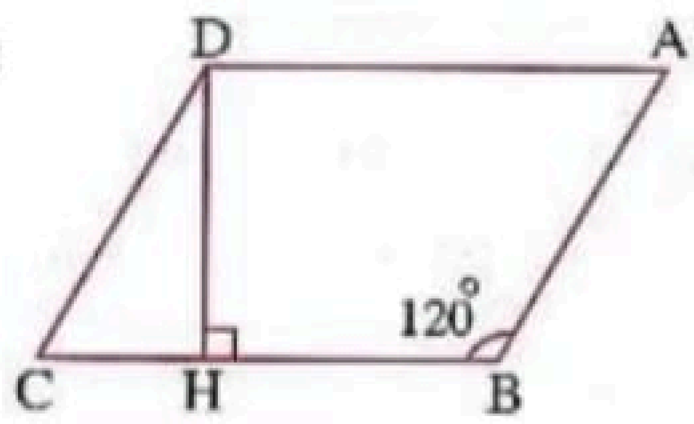
Proof ∵ $\overline{DA} \parallel \overline{BC}$
∴ m (∠ ADB) = m (∠ DBC) = 50 (alternate)
∵ ∠ DBA = ∠ ABE = $\frac{180 - 50}{2}$ = 65
∵ ∠ ABE = ∠ C = 65 (corresponding)
∴ $\overline{DC} \parallel \overline{AB}$, ∵ $\overline{DA} \parallel \overline{BC}$
∴ ABCD is a parallelogram

ABCD is a parallelogram in which :
m (∠ BAD) = 50° and BHOC is a parallelogram in which m (∠ BCO) = 60°, $L \in \overrightarrow{CB}$
Find with proof : m (∠ ABH)



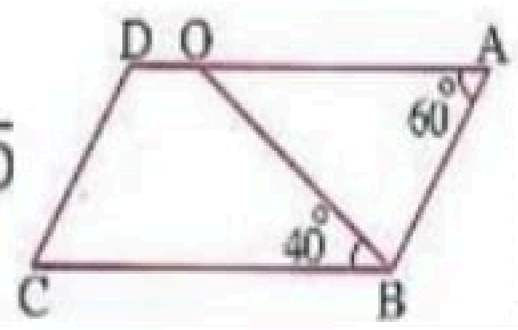
Proof ∵ ABCD is a ▱ ∠ A = 50°
∴ ∠ ABC = 180 − 50 = 130°
∵ BHOC is a ▱ ∵ ∠ BCO = 60°
∴ ∠ CBH = 180 − 60 = 120°
∵Sum of angles at the point B = 360°
∴ ∠ABH = 360 − (130 + 120) = 110°

ABCD is a parallelogram in which :
m (∠ B) = 120° and $\overline{DH} \perp \overline{BC}$
where $\overline{DH} \cap \overline{BC} = \{H\}$
Find : m (∠ HDC)



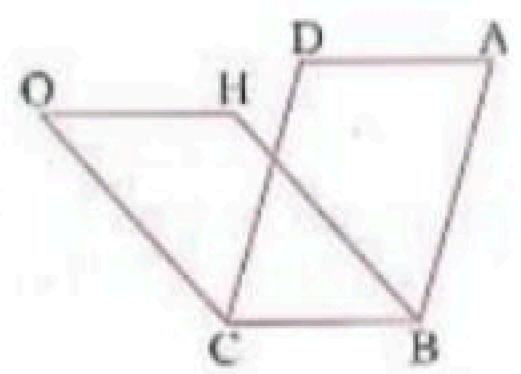
Proof ∵ ABCD is a parallelogram, ∠ B = 120°
∴∠ C = 180 − 120 = 60° , ∠ DHC = 90°
∵Sum of angles of Δ DHC = 180°
∴∠ HDC = 180 − (60 + 90) = 60°

ABCD is a parallelogram , in which :
m (∠ A) = 60° , m (∠ OBC) = 40° where $O \in \overline{AD}$
Find : m (∠ ABO)



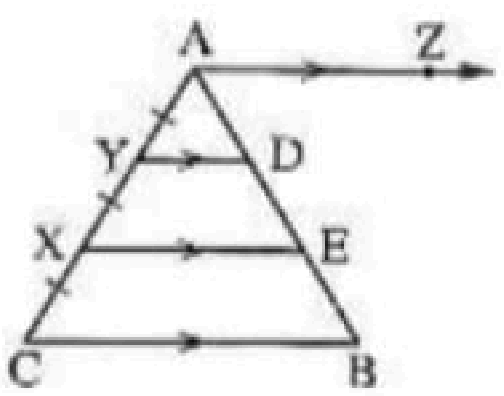
Proof ∵ ABCD is a parallelogram, ∠ A = 60°
∵ ∠ OBC = 40°
∴ ∠ ABO = 180 − (40 + 60) = 80°

Each of ABCD and HBCO is a parallelogram
Prove that : AD = HO



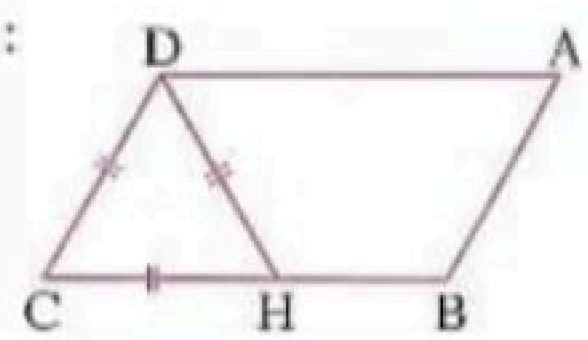
Proof ∵ABCD is a parallelogram
∴ DA = CB (1)
∵ HBCO is a parallelogram
∴ HO = CB (2)
From (1) , (2)∴ AD = HO

In the opposite figure :
$\overrightarrow{AZ} \parallel \overline{YD} \parallel \overline{XE} \parallel \overline{CB}$,
AY = YX = XC , AB = 18 cm.
Find the length of : $\overline{EB}$



Proof ∵ $\overrightarrow{AZ} \parallel \overline{YD} \parallel \overline{XE} \parallel \overline{CB}$
∵ AY = YX = XC
∴ AD = DE = EB = 18÷3 = 6 cm∴ EB = 6 cm

ABCD is a parallelogram , $H \in \overline{BC}$ where :
Δ DHC is an equilateral triangle.
(1) **Prove that :** HC = AB
(2) **Find :** m (∠ B) and m (∠ HDA)

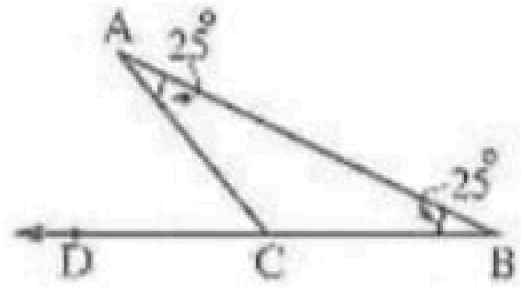


Proof ∵ ABCD is a ▱ ∴ AB = DC (1)
∵Δ DHC is an equilateral Δ
∴ CD = DH = HC (2)
From (1) , (2) ∴ HC = AB
∵ Each angle of Δ DHC = 60°
∴∠ B = 180 − 60 = 120°
∴ ∠HDA = 180 − (60 +60) = 60°

In the opposite figure :

m (∠ A) = m (∠ B) = 25°

Find : m (∠ ACD)



Proof ∵ ∠ ACD is exterior angle

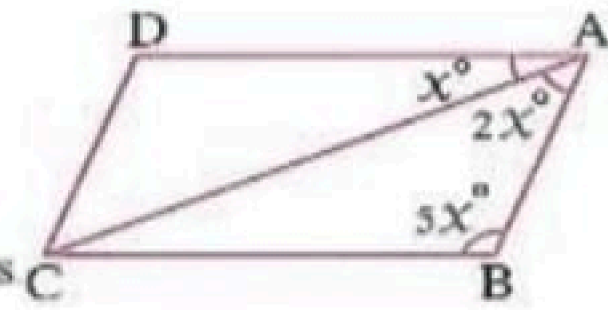
∴ ∠ACD = 25 + 25 = 50°

ABCD is a parallelogram in which :

m (∠ DAC) = x° , m (∠ BAC) = 2 x°

and m (∠ ABC) = 5 x°

Find : m (∠ BCD) and m (∠ ADC) in degrees



Proof ∵ ABCD is a parallelogram

∴∠ A + ∠ B = 180°

∴ 3 x + 5 x = 180°

∴ 8 x = 180° → ∴ x = 22.5°

∴∠ BCD = ∠ A = 3x = 3×22.5 = 67.5°

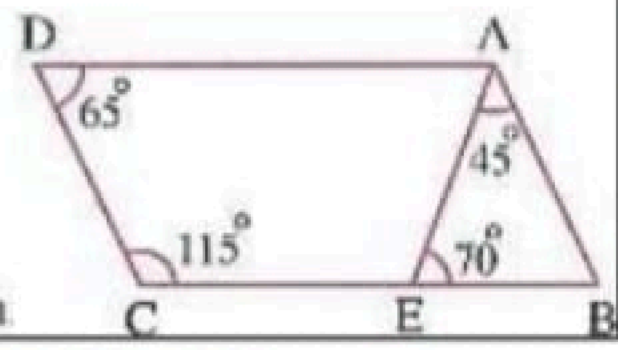
∴∠ ADC = ∠ B = 5x = 5×22.5 = 112.5°

E ∈ $\overline{BC}$, m (∠ BAE) = 45°,

m (∠ AEB) = 70° , m (∠ D) = 65°

and m (∠ C) = 115°

Prove that : ABCD is a parallelogram



Proof ∵ Sum of angles of Δ ABE = 180°

∴ ∠ B = 180 – (45 + 70) = 65°

∵∠ C + ∠ D = 180° ∴ $\overline{DA}$ // $\overline{CB}$ (1)

∵ ∠ C + ∠ B = 180° ∴ $\overline{DC}$ // $\overline{AB}$ (2)

From (1) , (2) ∴ ABCD is a parallelogram

Another Proof ∵ Sum of angles of Δ ABE = 180°

∴ ∠ B = 180 – (45 + 70) = 65°

∵ Sum of angles of the quadrilateral ABCD = 360°

∴ ∠ A = 360 – (65 + 115 + 65) = 115°

∴∠ D = ∠ B = 65° ∴ ∠ C = ∠ A = 115°

Each two opposite angles are equal

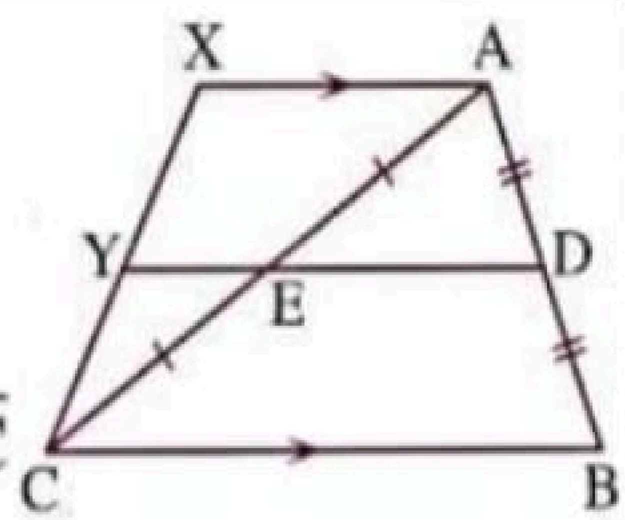
∴ ABCD is a parallelogram

In the opposite figure :

AD = DB , AE = EC ,

$\overline{AX}$ // $\overline{BC}$, $\overrightarrow{DE}$ ∩ $\overline{XC}$ = {Y}

Prove that : Y is the midpoint of $\overline{XC}$



Proof In Δ ABC

∵ D is the midpoint of $\overline{AB}$,

∵E is the midpoint of $\overline{AC}$

∴ $\overline{DE}$ // $\overline{CB}$

In ΔACX

∵E is the midpoint of $\overline{AC}$

∵ $\overline{YE}$ // $\overline{AX}$

∴ Y is the midpoint of $\overline{XC}$

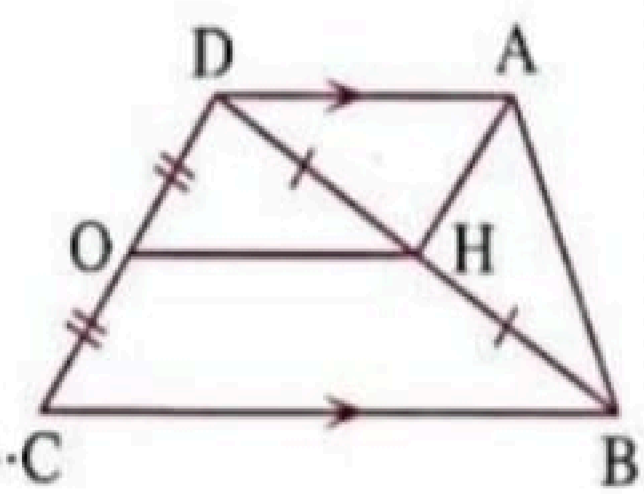
In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, AD = $\frac{1}{2}$ BC ,

H is the midpoint of $\overline{BD}$,

O is the midpoint of $\overline{CD}$

Prove that : AHOD is a parallelogram.



Proof In Δ DBC

∵ O is the midpoint of $\overline{DC}$,

∵H is the midpoint of $\overline{DB}$

∴ $\overline{OH}$ // $\overline{CB}$, OH = ½ BC (1)

∵ $\overline{AD}$ // $\overline{CB}$, AD = ½ BC (2)

From (1) , (2)

∴ $\overline{OH}$ // $\overline{DA}$, OH = AD

∴ AHOD is a parallelogram

In the opposite figure :

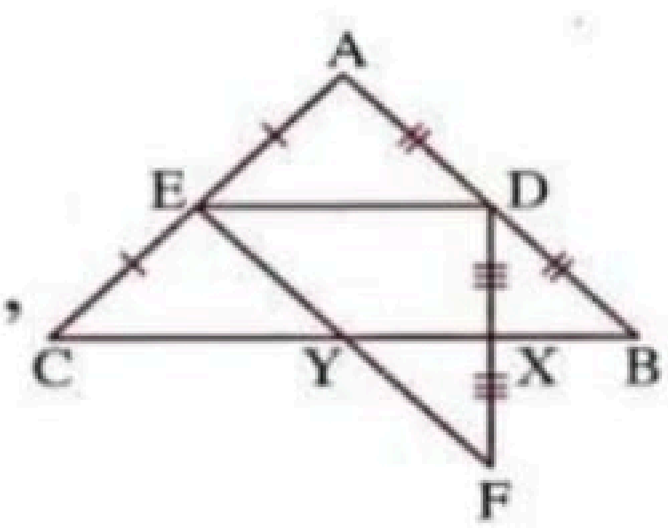
D is the midpoint of $\overline{AB}$,

E is the midpoint of $\overline{AC}$,

$\overline{DF}$ ∩ $\overline{BC}$ = {X}, DX = XF ,

BC = 12 cm.

Find the length of : $\overline{XY}$



Proof In Δ ABC

∵ D is the midpoint of $\overline{AB}$,

E is the midpoint of $\overline{AC}$

∴ DE = ½ BC ∴ DE = ½ × 12 = 6 cm

∵ $\overline{DE}$ // $\overline{CB}$

In Δ DFE

∵ X is the midpoint of $\overline{DF}$, $\overline{DE}$ // $\overline{XY}$

∴ Y is the midpoint of $\overline{EF}$

∴XY = ½ DE ∴XY = ½ × 6 = 3cm

- **Remark : The translation – the rotation – the reflection are reserves**

Length of the line segment – measures of the angles – parallelism

1) Choose

a) 180.

b) (3 ,1)

c) 120

d) Rhombus

e) 5

2) Complete

a) (2 , – 1)

b) (-2 , 1)

c) Two adjacent sides are equal Or two diagonals perpendicular

d) 180-60 = 120

e) (8 , 2)

3) Choose

a) (-3 , - 5)

b) 540

c) 2

d) 11

e) 45

f) (2 , 5)

g) 120

4) Complete

a) 360

b) MN =N – M

(5 , 1) –(2 , - 1) = (3,2)

A + T = A′

(2 , 3) + (3,2) = (5 , 5)

c) 180- 60 = 120

d) Rectangle

e) Bisects the third side

5) Complete

a) Two adjacent sides are equal Or two diagonals perpendicular

b) Equal

c) Y – axis

d) parallel to the third side and equal its half

e) (– 4 , - 1)

f) (– 3 , - 2)

g) Each two opposite sides are equal and parallel.

6) Choose

a) Rhombus

b) $\frac{(5-2)\times 180}{5}$ =108

c) Acute

d) (8+6)×2=28

e) (2 ,1)

7) Complete

a) parallel to the third side and equal its half

b) =

c) (2 , - 3)

10) Choose

a) (3 , 4)

b) 360

c) Square

d) (-5 ,-3)

e) $x^\circ = \frac{(n-2)\times 180}{n} = \frac{(8-2)\times 180}{8} = \dots$

f) The measure of the exterior angle of Δ equal to the sum of the measures of its non-adjacent interior angles

g) Square

h) 130

11) Complete

a) rhombus

b) right Δ

c) acute

d) (2 , 4)

e) (2 , 4)

f) (-2 , 2)

g) $(XY)^2 + (YZ)^2$

12) Choose

a) $x^\circ = \frac{(n-2)\times 180}{n} = \frac{(5-2)\times 180}{5} = \dots$

b) 2

c) Square

d) (2,1)

e) Square

f) Triangle

g) (-3 ,-2)

13) Choose

a) $x^\circ = \frac{(n-2)\times 180}{n} = \frac{(6-2)\times 180}{6} = \dots$

b) acute

c) equal

14) Complete

a) (5-2)x 180 =………

b) 120

c) Right =90

d) (-2 ,2)

e) -90 or 270

15) Choose

a) 45

b) 120

c) Right

d) 180

e) 360

16) Complete

a) half the length of the third side

b) Square

c) 5 cm

d) (3 ,-4) , (-3 , 4)

e) Point + Trans = image

(-1, 3) + Trans = (1 ,4)

Trans = (1 ,4) – (-1 , 3) = (0, 7)

Then (3, -2) + (0, 7) = (3, 5)